1. (10 pts) Complete the following.

\[ A = \{5, 20\} \quad B = \{5, 8\} \]
\[ C = \{5, 10, 20, 40, \ldots\} \quad D = \{5, 10, 15, 20, 25, \ldots\} \]

(a) \( C \cap D = \) 
(b) \( A \times B = \) 
(c) \( C \cup D = \) 
(d) Give 3 elements in \( 2^{A \times B} \) 
(e) \( A \times B = B \times A \) (true or false?)

2. (8 pts) Complete the following.

\[ L_1 = \{a^n b^n \mid n > 0\} \]
\[ L_2 = \{b^n a^m \mid n > m > 0\} \]
\[ L_3 = \{ w \in \Sigma^* \mid n_a(w) = n_b(w) \}, \Sigma = \{a, b\} \]

(a) \( L_1 \circ L_2 = \) 
(b) \( L_1 \cap L_2 = \) 
(c) \( L_3 \cap a^\ast b^\ast a^\ast = \) 
(d) \( L_1 \cup \{b^n a^n \mid n > 0\} = L_3 \) (TRUE or FALSE?)

3. (12 pts) Answer TRUE or FALSE to each of the statements below.

(a) Given a DFA \( M \), there exists a nonambiguous CFG \( G \) such that \( L(M) = L(G) \). (TRUE or FALSE?)
(b) Given an NFA \( M \) with 100 final states, there exist a regular expression \( r \) such that \( L(M) = L(r) \). (TRUE or FALSE?)
(c) Given a regular expression \( r \) with 4 terminals and at most 4 operators (for example, \( a b^* c + c \) has 4 terminals and 4 operators(\( \circ, \circ, \circ, +\)), there exist a DFA \( M \) with 4 or fewer states such that \( L(M) = L(r) \). (TRUE or FALSE?)
(d) \( L = \{a^n b^m c^n \mid 0 \leq m < 5, n > 100\} \). \( L \) is regular. (TRUE or FALSE?)
(e) \( L = \{a^{2n} b^{2m} c^{3p} \mid n > 0, m > 0, p > 0\} \). \( L \) is regular. (TRUE or FALSE?)
(f) \( L = \{a^n b^m \mid m < n, n > 0, 0 \leq m < 100\} \). \( L \) is regular. (TRUE or FALSE?)

4. (12 pts) Draw a DFA (not an NFA!) for the following languages. Do not show trap states. (show the transition diagram, indicate the start state by a short arrow, and final states by double circles.)
(a) \( L = \{ w \in \Sigma^* \mid w \) has at least one \( a \) and \( bba \) is not a substring of \( w \} \), \( \Sigma = \{a, b\} \).
(b) \( L = \{ w \in \Sigma^* \mid aaa \) is not a substring of \( w \) and \( n_b(w) \mod 3 = 0 \} \), \( \Sigma = \{a, b\} \).

5. (8 pts) Convert the following NFA to a DFA using the algorithm discussed in class.

6. (8 pts) Convert the following DFA to a minimum state DFA. Show the tree distinguishing the states and briefly explain at each level the reason for distinguishing the states. Show the resulting minimal DFA with states labeled with names from their original states (for example, combined states A and B would be called state AB).
7. (6 pts) Consider the following DFA M. Give a regular expression R such that $L(R) = L(M)$.

8. (6 points) Consider the following language.

$L = \{w \in \Sigma^* \mid n_a(w) \text{ is even and there is at least one } b\}$

Write a regular grammar $G$ for $L$ such that $L = L(G)$.

**Pumping Lemma:** Let $L$ be an infinite regular language. $\exists$ a constant $m > 0$ such that any $w \in L$ with $|w| \geq m$ can be decomposed into three parts as $w = xyz$ with

- $|xy| \leq m$
- $|y| \geq 1$
- $xy^iz \in L$ for all $i \geq 0$
9. (8 pts) Use the Pumping Lemma to prove
   \[ \Sigma = \{a, b\}, L = \{w \in \Sigma^* | n_b(w) > 2 \ast n_a(w)\} \]
   is not regular.

   **Proof:** (SHOW ALL STEPS! Some have been started for you.)
   Assume
   Choose \( w = \)

10. (6 points) Consider the following language. \[ L = \{a^n b^p c^m | n > p + m, p > 0, m \geq 0\} \]
    Write a **context-free grammar** for \( L \).

11. (6 points) Show the following grammar is ambiguous.
    \[
    \begin{align*}
    S & \rightarrow aAb | aS \\
    A & \rightarrow Abb | aB \\
    B & \rightarrow a | Bb
    \end{align*}
    \]

12. (10 pts) Consider the following property, SwapFirstaWithLastb (SwapFL). If \( L \) is a regular language, then
    \[ \text{SwapFL}(L) = \{w = vbxaz | vaxbz \in L, v \in (\Sigma - \{a\})^*, z \in (\Sigma - \{b\})^*, x \in \Sigma^*\} \]
    with \( \Sigma = \{a, b\} \). In other words, SwapFL(L) accepts a word from L with the first a and last b swapped. For example, if \( aababa \in L \), then \( babaaaa \in \text{SwapFL}(L) \). If \( babab \in L \), then \( bbbaa \in \text{SwapFL}(L) \).
    **Prove** that the regular languages are closed under the SwapFL(L) property. (Show all steps!)