NOTE: \( n_a(w) \) means the number of a’s in the string \( w \).
(h) \( L = \{a^n b^m \mid m > 10, n > 20\} \). \( L \) is regular. (TRUE or FALSE?)

(i) Consider the proof showing that if \( M \) is an NPDA, then there exists a CFG \( G \) such that \( L(M) = L(G) \). In this proof \((q_j, Aq_k)\) means that there is a transition from state \( q_j \) to state \( q_k \) in which the stack is exactly the same except \( A \) is popped off the stack. (TRUE or FALSE?)

(j) In the DFA to minimal state DFA algorithm, if two states are “distinguishable on \( a^* \)”, then one of those states has an \( a \) transition to a final state and the other such state has an \( a \) transition to a nonfinal state. (TRUE or FALSE?)

5. (8 pts) Draw a DFA for the following language. Do not show trap states. (show the transition diagram, indicate the start state by a short arrow, and final states by double circles.)

\( L = \{w \in \Sigma^* \mid n_a(w) \mod 3 = 2 \text{ and the number of } b\text{'s is odd} \} \Sigma = \{a, b\} \).

For example, \( bbaba, aba \) and \( abbabbababab \) are in \( L \).

6. (3 pts) What is the language of the following regular grammar?

\[
\begin{align*}
S & \rightarrow aaA \mid bB \\
A & \rightarrow aaS \mid bB \\
B & \rightarrow bC \mid \lambda \\
C & \rightarrow bB
\end{align*}
\]

7. (6 pts) Write a context-free grammar for the following language.

\( L = \{a^n b^m c^n \mid n > 0, m > 0 \text{ and the number of } b\text{'s are even} \} \)

8. (6 pts) Consider the two labeled finite automaton below. Using the algorithm from class, build a new finite automaton that represents the intersection of the two finite automaton.

![Diagram of two finite automata](image-url)
9. (10 pts) Consider \( L = \{ b^n a^m \mid 0 < n \leq m \leq 2n \} \). Draw the transition diagram for a nondeterministic pushdown automaton \( M \) that accepts \( L \) by final state. (Remember to identify the start state by an arrow and final states by double circles. Format of labels are \( a, b; cd \) where \( a \) is the symbol on the tape, \( b \) is the symbol on top of the stack that is popped, and \( cd \) are pushed onto the stack (with \( c \) on top of \( d \)). \( Z \) is on top of the stack when \( M \) starts. )

(a) First list 3 strings in \( L \).

(b) Now draw the transition diagram.

10. (6 pts) **Pumping Lemma:** Let \( L \) be an infinite regular language. \( \exists \) a constant \( m > 0 \) such that any \( w \in L \) with \( |w| \geq m \) can be decomposed into three parts as \( w = xyz \) with

\[
|xy| \leq m \\
|y| \geq 1 \\
xy^iz \in L \text{ for all } i \geq 0
\]

**Use the Pumping Lemma to prove**

\( L = \{ w \in \Sigma^* \mid n_a(w) \geq 3 \cdot n_b(w) \} \) is not regular. \( \Sigma = \{a, b\} \).

**Proof:** (SHOW ALL STEPS! Some have been started for you.)

Assume ________________________________

Choose \( w = ________________________________ \)

11. (8 pts) Consider the following property, Replace Every Second a With b (Re2awb). If \( L \) is a regular language, then show that Re2awb(L) is a regular language.

For example, if \( aabbabaa \) is in \( L \), then \( abbbabba \) (every second \( a \) is replaced with \( b \)) is in Re2awb(L). If \( bbb \) is in \( L \), then \( bbb \) is in Re2awb(L) (no \( a \)'s to replace).