1. (10 pts) Consider the following languages. Write “REG” if it is regular, “CFL” if it is a CFL and not regular, and write “NOT” if it is not a CFL.

(a) \(L = \{a^n b^m c^n \mid n > 0, m > 0, \text{ and } m \text{ is a multiple of } 3\}, \Sigma = \{a, b, c\}. \) \text{ CFL}

(b) \(L = \{a^{2n} b^m \mid n > 0, m > 0\}, \Sigma = \{a, b\}. \) \text{ REG}

(c) \(L = \{a^n b^m c^d \mid p + m > 2 \ast (n + t), n > 0, m > 0, p > 0, t > 0\}, \Sigma = \{a, b, c, d\}. \) \text{ CFL}

(d) \(L = \{w \in \Sigma^* \mid n_c(w) > 2 \ast n_b(w) > n_c(w)\}, \Sigma = \{a, b, c\}. \) \text{ NOT}

(e) \(L = \{a^n b^{2n} c^m \mid m > n \text{ and } 0 < n < 100\}, \Sigma = \{a, b, c\}. \) \text{ REG}

2. (12 pts) Answer TRUE or FALSE to each of the statements below.

(a) If \(M\) is an NPDA, then there exists a TM \(M'\) such that \(L(M) = L(M')\) \(\) (TRUE or FALSE?)

(b) If \(G\) is a CFG with only 1 variable in each right hand side of a rule, then \(G\) is regular grammar. (TRUE or FALSE?)

(c) If \(M_1\) is a DFA and \(M_2\) is an NPDA, then \(L = L(M_1) \cap L(M_2)\) is a CFL. (TRUE or FALSE?)

(d) If \(G\) is a CFG with \(\lambda\) not in \(L(G)\), then there exists a grammar \(G'\) such that \(L(G) = L(G')\) and there are no \(\lambda\) rules in \(G'\). (TRUE or FALSE?)

(e) There are no shift/shift conflicts in an LR(1) parse table. (TRUE or FALSE?)

(f) The first “L” in LL(1) parsing and the “L” in LR(1) parsing represent the same thing. (TRUE or FALSE?)
3. (3 pts) Give an example of a CFG that is not LL(1). Explain why it is not LL(1).

\[ S \rightarrow aSa \]

you need 2 lookaheads to decide which rule to use.

4. (3 pts) What are the differences between a Turing machine and a finite state machine with respect to the tape and the tape head? Explain.

IN FSA, the tape is read only and the tape head only moves right.
IN TM, the tape is read/write and the tape head moves left or right.

5. (3 pts) Assume the tape alphabet is \{a, b, B\} where B represents a blank square on the tape. Show the Turing Machine transition diagram for the Building Block R_a. (R_a means move right once then keep moving right until the tape head reaches an a).

6. (3 pts) The following grammar is LL(k) for what value of k?

\[ S \rightarrow aBaa \mid Acd \\
A \rightarrow aA \mid b \\
B \rightarrow aabB \mid b \]

7. (3 pts) Context-free languages are not closed under intersection. Give an example of two different CFL's L1 and L2 such that L1 \( \cap \) L2 is a CFL.

\[ L_1 = \{ a^n b^n \mid n \geq m, m \geq 5 \} \text{ is CFL} \]
\[ L_2 = \{ a^n b^n \mid n \leq m, n \geq 5 \} \text{ is CFL} \]
\[ L_1 \cap L_2 = \{ a^n b^n \mid n > 5 \} \text{ is CFL} \]
8. (10 pts) Consider the following grammar (DO NOT change the grammar):

\[
S \rightarrow ABc \\
A \rightarrow aSBa \mid \lambda \\
B \rightarrow bB \mid dAd \mid \lambda
\]

For this problem you will construct the LL(1) parse table.

(a) Calculate FIRST and FOLLOW for the variables in the grammar.

<table>
<thead>
<tr>
<th></th>
<th>FIRST</th>
<th>FOLLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>a, b, c, d</td>
<td>$, b, d, a</td>
</tr>
<tr>
<td>A</td>
<td>a, \lambda</td>
<td>d, b, c</td>
</tr>
<tr>
<td>B</td>
<td>b, d, \lambda</td>
<td>a, c</td>
</tr>
</tbody>
</table>

(b) Calculate all entries in the LL(1) Parse Table. If there are multiple rules for an entry, write in all the rules.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>$</th>
</tr>
</thead>
</table>
| S   | ABC | ABC | ABC | ABC | $
| A   | aSBa | \lambda | \lambda | \lambda | $
| B   | \lambda | bB | \lambda | dAd | $

(c) Is this grammar an LL(1) grammar? Explain. Yes, there are no conflicts in the table.
9. (15 pts) Construct the LR parsing table for the following grammar (DO NOT change the grammar.) A new start symbol S' and production have already been added to the grammar.

1) S' → S
2) S → aAB
3) A → aB
5) B → b
4) A → λ

(a) Calculate the FIRST and FOLLOW sets of variables.

<table>
<thead>
<tr>
<th></th>
<th>FIRST</th>
<th>FOLLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>a</td>
<td>$</td>
</tr>
<tr>
<td>A</td>
<td>a, λ</td>
<td>b</td>
</tr>
<tr>
<td>B</td>
<td>b</td>
<td>b, $</td>
</tr>
</tbody>
</table>

(b) Construct the transition diagram of the DFA that models the stack. Number the states, show marked productions, and identify final states by two circles.
(c) Construct the LR parse table that corresponds to the transition diagram drawn in part b. (Note: all the rows and columns given may not be needed. If there are multiple items for an entry, write all in the entry.)

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>$</th>
<th>S</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<tr>
<td>1</td>
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<td>acc</td>
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<td>2</td>
<td>$4 r4</td>
<td>3</td>
<td></td>
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<td>3</td>
<td>$6</td>
<td>5</td>
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<td>r5 r5</td>
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</tbody>
</table>
10. (8 pts) Consider the following L-system.

Axiom: \( g + X \)
\( X \rightarrow g + Y g X \)
\( Y \rightarrow g \{ - g f g \} + g \)

angle 90
color black
lineWidth 2
distance 10

Recall that \( g \) is for drawing a line, \( f \) is for moving forward, \( + \) means change the direction by the angle clockwise, \( - \) means change the direction by the angle counterclockwise and \( [ \) and \( ] \) are used for stacking operations.

Assume a \( g \) drawn with distance 10 and lineWidth 2 is about this size

a. Render the L-system and draw the axiom if there is a visual picture for it.

\[
\]

b. Give the first string in the language (after the axiom) and draw it.

\[
g + g + Y g X
\]

\[
\]

b. Give the second string in the language and draw it.

\[
g + g + g \{ - g f g \} + g g g + Y g X
\]

\[
\]
11. (6 pts) **Pumping Lemma for CFL’s** Let \( L \) be any infinite CFL. Then there is a constant \( m \) depending only on \( L \), such that for every string \( w \) in \( L \), with \( |w| \geq m \), we may partition \( w = uvxyz \) such that:

\[
|vxy| \leq m, \text{ (limit on size of substring)} \\
|vy| \geq 1, \text{ (} u \text{ and } y \text{ not both empty)} \\
\text{For all } i \geq 0, uv^i xy^i z \in L
\]

Consider \( L = \{a^n b^n c^n | n > 0, m > n \} \) \( \Sigma = \{a, b, c\} \).

Prove \( L \) is not a context-free language.

You only have to fill in the parts below. Assume \( L \) is a context-free language.

(a) Choose \( w = a^m b^{m+1} c^{2m} \)

(b) Prove the case when \( v = a^{t_1} \) and \( y = b^{t_2} \) (both are strings of \( a \)'s)

\[
i = 0 \\
\text{since } t_1 + t_2 > 0 \\
\text{if } t_1 \neq 0 \text{ then } 2 \times na \neq nc \\
\text{if } t_1 = 0 \text{ then } nb \leq na
\]

(c) Prove the case when \( v = b^{t_1} \) and \( y = b^{t_3} \)

\[
i = 0 \\
\text{since } nb \leq na
\]
12. (9 pts) Construct a one-tape TM transducer (using a transition diagram) that represents binary subtraction of 1. Assume \( w > 1 \) and \( w \) is a binary number. \( f(w) = w - 1 \).

For example, \( f(101) = 100, f(1000) = 111 \)

In drawing the transition diagram, remember to identify the start state by an arrow and final states by double circles. Format of labels are \( a; b, R \) where \( a \) is the symbol read on the tape, \( b \) is the symbol written to the tape and \( R \) is the direction moved (you can use \( L \) and \( R \) for directions.) Make sure the tape head is pointing to the leftmost symbol of the output.

Assume \( |w| = n \). What is the worst case running time (big-Oh) of your TM?
13. (9 pts) Construct a TM (using building blocks) for the following function. \( w \in \{a, b\}^* \), 
\( f(w) = w' \) where \( w' \) is \( w \) with all but the first \( a \) removed.
For example, \( f(bbaaba) = (bbab) \) and \( f(aaaba) = ab \).
See the building block notation on the next page.

Assume \( |w| = n \). What is the running time in terms of \( n \) (big-Oh) of your TM? \( O(n^2) \)
Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

$z$ is any symbol in $\Gamma$

$x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right
3. $L$ - move left
4. $x$ - write $x$ (and don’t move)
5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an $a$
7. $R_{\neg a}$ - move right until you see anything that is not an $a$
8. $L_{\neg a}$ - move left until you see anything that is not an $a$
9. $h$ - halt in a final state
10. $a, b \rightarrow w$

If the current symbol is $a$ or $b$, let $w$ represent the current symbol.