1. (10 pts) Consider the following languages. Write “REG” if it is regular, “CFL” if it is a CFL and not regular, and write “NOT” if it is not a CFL.

   (a) \( L = \{ a^{3n}b^{2n}c^{n}d^{4n} \mid n > 0 \} \)  

   (b) \( L = \{ w \in \Sigma^* \mid n_a(w) > n_b(w), n_c(w) \mod 6 = 0 \}, \Sigma = \{a, b, c\} \)  

   (c) \( L = \{ a^n b^n c^n d^n \mid n + p = m + q, n > 0, m > 0, p > 0, q > 0 \} \)  

   (d) \( L = \{ a^n b^n c^n \mid 0 < n < 100, m > 100 \} \)  

   (e) \( L = \{ w \in \Sigma^* \mid n_b(w) = (n_a(w) \mod 3) \text{ and } n_c(w) = n_a(w) + n_b(w) \}, \Sigma = \{a, b, c\} \)

2. (6 pts) Answer TRUE or FALSE to each of the statements below.

   (a) If \( M \) is an NPDA, then there exists a TM \( M' \) such that \( L(M) = L(M') \) and \( M' \) runs twice as fast as \( M \). (TRUE or FALSE?)

   (b) An NPDA will always halt no matter what the input. (TRUE or FALSE?)

   (c) The intersection of a language represented by an NPDA and a language represented by an NFA is a CFL. (TRUE or FALSE?)

3. (2 pts) What is the key difference between a Turing machine and a modern day personal computer, such as an IBM PC?

4. (2 pts) A TM is defined by \( M = (Q, \Sigma, \Gamma, \delta, q_0, B, F) \). Give the formal definition of \( \delta \).

5. (2 pts) List the 4 actions of an LR parser.

6. (2 pts) Give a CFG with 5 rules or less such that one of the rules is a unit production and one of the rules is a useless production. Identify the unit and useless productions.
7. (2 pts) Give a CFG that is LL(3) and has 5 rules or less.

8. (12 pts) Consider \( L = \{a^n b^m c^p d^r \mid a + r = m + p, n > 0, m > 0, p > 0, r > 0\} \). Draw the transition diagram for a nondeterministic pushdown automaton \( M \) that accepts \( L \) by final state. (Remember to identify the start state by an arrow and final states by double circles. Format of labels are \( a; b; cd \) where \( a \) is the symbol on the tape, \( b \) is the symbol on top of the stack that is popped, and \( cd \) are pushed onto the stack (with \( c \) on top of \( d \)). \( Z \) is on top of the stack when \( M \) starts.)

(a) First list 3 strings in \( L \).

(b) Now draw the transition diagram.

9. (12 pts) Consider the following grammar (DO NOT change the grammar):

\[
\begin{align*}
S & \rightarrow aABd \\
A & \rightarrow aA \mid Bc \\
B & \rightarrow bBa \mid \lambda
\end{align*}
\]

For this problem you will construct the LL(1) parse table.

(a) Calculate FIRST and FOLLOW for the variables in the grammar.

<table>
<thead>
<tr>
<th>FIRST</th>
<th>FOLLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>

(b) Calculate all entries in the LL(1) Parse Table. If there are multiple rules for an entry, write in all the rules.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) This language is LL(k) where \( k = ? \)
10. (18 pts) Construct the LR parsing table for the following grammar (DO NOT change
the grammar.) A new start symbol S’ and production have already been added to the
grammar.

1) S’ → S  
2) S → bBA  
3) A → Bc  
4) A → aA  
5) A → a  
6) B → λ

(a) Calculate the FIRST and FOLLOW sets of variables.

<table>
<thead>
<tr>
<th></th>
<th>FIRST</th>
<th>FOLLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Construct the transition diagram of the DFA that models the stack. Number the
states, show marked productions, and identify final states by two circles.

(c) Construct the LR parse table that corresponds to the transition diagram drawn
in part b. (Note: all the rows and columns given may not be needed. If there are
multiple items for an entry, put both.)
11. **(10 pts) Pumping Lemma for CFL’s** Let $L$ be any infinite CFL. Then there is a constant $m$ depending only on $L$, such that for every string $w$ in $L$, with $|w| \geq m$, we may partition $w = uvxyz$ such that:

$|vxy| \leq m$, (limit on size of substring)  
$|vy| \geq 1$, ($v$ and $y$ not both empty)  
For all $i \geq 0$, $uw^ixy^iz \in L$

Prove that $L = \{a^n b^p c^n | 0 < p < n \}$ is not a context-free language.

You only have to fill in the parts below. Assume $L$ is a context-free language.

(a) Choose $w =$
(b) Prove the case when $v = a^{t_1}$ and $y = a^{t_2}$ (both are strings of a’s)
(c) Prove the case when $v = a^{t_1}$ and $y = b^{t_3}$
(d) Prove the case when \( v = b^t_1 \) and \( y = b^t_2 \)
(e) Prove the case when \( v = b^t_1 \) and \( y = c^t_3 \)

12. (11 pts) Construct a TM (using a transition diagram) for the following function. \( f(x) = y \), where \( \Sigma = \{1\} \), \( x \in \Sigma^+ \) and represents a unary number, and \( y \) is \( x \) divided by 3 using integer division.

For example, \( f(11111111) = 11 \) since \( 8/3 = 2 \), and \( f(111111) = 11 \) since \( 6/3 = 2 \).

In drawing the transition diagram, remember to identify the start state by an arrow and final states by double circles. Format of labels are \( a; b; R \) where \( a \) is the symbol read on the tape, \( b \) is the symbol written to the tape and \( R \) is the direction moved (you can use \( L \) and \( R \) for directions.) Make sure the tape head is pointing to the leftmost symbol of the output.

Assume \( |w| = n \). What is the worst case running time (big-Oh) of your Turing machine?

**BUILDING BLOCK NOTATION**

(a) \( s \) - start
(b) \( R \) - move right one cell
(c) \( L \) - move left one cell
(d) \( x \) - write \( x \) (and don’t move)
(e) \( R_a \) - move right until you see an \( a \) (always moves at least one cell, ignores the symbol on the cell the tape head starts at)
(f) \( L_a \) - move left until you see an \( a \)
(g) \( R_{-a} \) - move right until you see anything that is not an \( a \)
(h) \( L_{-a} \) - move left until you see anything that is not an \( a \)
(i) \( R_{ab} \) - move right until you see an \( a \) or a \( b \)
(j) \( L_{ab} \) - move left until you see an \( a \) or a \( b \)
(k) \( h \) - halt in a final state

(l) \( \xrightarrow{a,b} \) \( w \)

If the current symbol is \( a \) or \( b \), let \( w \) represent the current symbol.

(m) \( C \) - makes a copy of the input with a 0 between the input and the copy.

(n) \( S_L \) - shift all symbols (up to a blank) that are to the right of the tape head, shift them to the left one square.

(o) \( S_R \) - shift all symbols (up to a blank) that are to the left of the tape head, shift them to the right one square.

13. (11) Construct a TM (using building blocks) for the following function. \( f(w) = a^n b^n \) where \( w \in \Sigma^* \), \( \Sigma = \{a, b\} \). You can use any of the building block notation on the previous page (note that the \( | \) symbol (or) has been added as rules (i) and (j)).
For example, $f(abbaa)$ writes $aaabb$ to the tape, and $f(bbbaa)$ writes $aabb$ to the tape. The answer must be surrounded by blanks. It is ok to have other stuff still on the tape when the Turing machine halts.

Note $|w| = n + m$. What is the worst case running time (big-Oh) of your Turing machine?