CPS 140 Exam 2 Spring 2005

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NOTE: \( n_a(w) \) means the number of a’s in the string \( w \).

1. (10 pts) Consider the following languages. Write “REG” if it is regular, “CFL” if it is a CFL and not regular, and write “NOT” if it is not a CFL.

   (a) \( L = \{ a^n b^n c^n d^n | n > 0 \} \)  
   (b) \( L = \{ a^n b^m c^n d^m | n > m, 0 < m < 50 \} \)  
   (c) \( L = \{ a^n b^m | n > 0, m > 0, n + m > 200 \} \)  
   (d) \( L = \{ a^{2n} b^{2m} | n > m > 0 \} \)  
   (e) \( L = \{ wa^{2n} w^R | w \in \Sigma^* \}, \Sigma = \{ a, b, c \} \), and \( w^R \) is the reverse of \( w \).

2. (12 pts) Answer TRUE or FALSE to each of the statements below.

   (a) If a grammar can result in building an LR(1) parse table with no conflicts, then the grammar will result in building an LL(1) parse table with no conflicts. (TRUE or FALSE?)

   (b) If an NPDA had 2 stacks instead of 1 stack, then \( a^n b^n c^n \) could be recognized on this 2-stack NPDA. (TRUE or FALSE?)

   (c) If a language \( L \) is a CFL, then there exists a TM \( M \) such that \( L = L(M) \). (TRUE or FALSE?)

   (d) If \( G_1 \) and \( G_2 \) are both CFG’s, then \( L(G_1) \cap L(G_2) \) is a CFL. (TRUE or FALSE?)

   (e) \( \lambda \) can be in a FOLLOW set. (TRUE or FALSE?)

   (f) Removing useless productions from a CFG can result in adding in unit-productions. (TRUE or FALSE?)

3. (3 pts) Circle the rules that are left-recursive.

   \( S \rightarrow aSb \quad S \rightarrow ABb \quad A \rightarrow Aaa \)
   \( A \rightarrow a \quad B \rightarrow baB \quad B \rightarrow \lambda \)

4. (3 pts) For each of the following grammars, identify them as CFG, CNF, GNF or None. Give all labelings that apply.

   \( S \rightarrow ABC \quad A \rightarrow a \quad B \rightarrow b \quad C \rightarrow c \)
\[
S \rightarrow aBC \\
B \rightarrow bB \mid b \\
C \rightarrow c
\]

\[
S \rightarrow ABC \\
AB \rightarrow BA \\
AC \rightarrow c \\
B \rightarrow b
\]

5. (3 pts) What do the \(L\), \(L\) and \(k\) stand for in LL\((k)\) parsing?

6. (3 pts) Which one of the following labels could be on a transition in an NPDA that was formed by converting a CFG to an NPDA using the LL parsing method.

\[a, aSb; S \quad \lambda, S; aSb \quad \lambda, aSb; S \quad a, S; aSb\]

7. (3 pts) Consider the LR parsing process. How many arcs should be outgoing from a state in the DFA that has the following item set?

\[
S \rightarrow aSb \\
A \rightarrow aa_{aA} \\
B \rightarrow Ba_\_ \\
S \rightarrow _{aSb} \\
S \rightarrow c
\]

8. (3 pts) The following grammar is LL\((k)\) for what value of \(k\)?

\[
S \rightarrow ABcS \mid aab \\
A \rightarrow aA \mid a \\
B \rightarrow \_bc
\]

9. (6 pts) **Pumping Lemma for CFL’s** Let \(L\) be any infinite CFL. Then there is a constant \(m\) depending only on \(L\), such that for every string \(w\) in \(L\), with \(|w| \geq m\), we may partition \(w = uvxyz\) such that:

\[
|vxy| \leq m, \text{ (limit on size of substring)} \\
|vy| \geq 1, \text{ (v and y not both empty)} \\
\text{For all } i \geq 0, \ uv^ixy^iz \in L
\]

Prove that \(L = \{a^n b^s c^p \mid n = s, p > s > 0\}\), is not a context-free language.

You only have to fill in the parts below. Assume \(L\) is a context-free language.

(a) Choose \(w = \)
(b) Prove the case when \( v = a^{t_1} \) and \( y = a^{t_2} \) (both are strings of a’s)
(c) Prove the case when \( v = a^{t_1} \) and \( y = b^{t_3} \) and there must be a’s and b’s (thus \( t_1 > 0 \) AND \( t_3 > 0 \), when either one is 0, another case is formed)

10. (10 pts) Consider the following grammar (DO NOT change the grammar):

\[
S \rightarrow abBS \mid Ab \\
A \rightarrow c \mid \lambda \\
B \rightarrow AAd
\]

For this problem you will construct the LL(1) parse table.

(a) Calculate FIRST and FOLLOW for the variables in the grammar.

<table>
<thead>
<tr>
<th></th>
<th>FIRST</th>
<th>FOLLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Calculate all entries in the LL(1) Parse Table. If there are multiple rules for an entry, write in all the rules.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>A</td>
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<tr>
<td>B</td>
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</tbody>
</table>

11. (16 pts) Construct the LR parsing table for the following grammar (DO NOT change the grammar.) A new start symbol \( S’ \) and production have already been added to the grammar.

1) \( S’ \rightarrow S \)  
2) \( S \rightarrow aSB \)  
3) \( S \rightarrow Aa \)  
4) \( A \rightarrow \lambda \)  
5) \( B \rightarrow b \)

(a) Calculate the FIRST and FOLLOW sets of variables.
(b) Construct the transition diagram of the DFA that models the stack. Number the states, show marked productions, and identify final states by two circles.

(c) Construct the LR parse table that corresponds to the transition diagram drawn in part b. (Note: all the rows and columns given may not be needed. If there are multiple items for an entry, write all in the entry.)

12. (9 pts) Construct a TM (using a transition diagram) for the following function. \( f(w) = w + 1 \), where \( \Sigma = \{0, 1\} \), \( w \in \Sigma^* \), \( w \) represents a binary number, so \( f(w) \) is adding one to a binary number.

For example, \( f(101) = 110 \), \( f(111) = 1000 \), \( f(100) = 101 \), and \( f(0) = 1 \).
In drawing the transition diagram, remember to identify the start state by an arrow and final states by double circles. Format of labels are $a; b, R$ where $a$ is the symbol read on the tape, $b$ is the symbol written to the tape and $R$ is the direction moved (you can use $L$ and $R$ for directions.) Make sure the tape head is pointing to the leftmost symbol of the output. Assume $|w| = n$. What is the worst case running time (big-Oh) of your TM?

13. (9 pts) Construct a TM (using building blocks) for accepting the following language $L = \{ a^n b^m \mid m > 2n, n > 0$ and $n$ is even $\}$. See the building block notation on the next page. What is the running time in terms of $n$ and $m$ (big-Oh) of your TM?

Notation for Simplifying Turing Machines

Suppose $\Gamma = \{ a, b, c, B \}$

$z$ is any symbol in $\Gamma$

$x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right
3. $L$ - move left
4. $x$ - write $x$ (and don’t move)
5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an $a$
7. $R_{\neg a}$ - move right until you see anything that is not an $a$
8. $L_{\neg a}$ - move left until you see anything that is not an $a$
9. $h$ - halt in a final state
10. $\overset{a,b}{\rightarrow} \overset{w}{\rightarrow}$

If the current symbol is $a$ or $b$, let $w$ represent the current symbol.