Problem 2.1-2: Show that for any real constants a and b, $b > 0$,

$$(n + a)^b = \Theta(n^b)$$

To show $f(n) = \Theta(g(n))$, we must show O and Ω. *Go back to the definition!*

- *Big O* – Must show that $(n + a)^b \leq c_1 \cdot n^b$ for all $n > n_0$. When is this true? If $c_1 = 2$, this is true for all $n > |a|$ since $n + a < 2n$, and raise both sides to the b.

- *Big Ω* – Must show that $(n + a)^b \geq c_2 \cdot n^b$ for all $n > n_0$. When is this true? If $c_2 = 1/2$, this is true for all $n > |a|$ since $n + a > n/2$, and raise both sides to the b.

Note the need for absolute values.

Problem 2.1-4:

(a) Is $2^{n+1} = O(2^n)$?

(b) Is $2^{2n} = O(2^n)$?

(a) Is $2^{n+1} = O(2^n)$?

Is $2^{n+1} \leq c * 2^n$?

Yes, if $c \geq 2$ for all n

(b) Is $2^{2n} = O(2^n)$?

Is $2^{2n} \leq c * 2n$?

note $2^{2n} = 2^n * 2^n$

Is $2^n * 2^n \leq c * 2^n$?

Is $2^n \leq c$?

No! Certainly for any constant c we can find an n such that this is not true.