Due Date: October 27, 2005

Problem 1: Let R be a set of n rectangles in the plane. Describe an algorithm that reports all k pairs of intersecting rectangles in time $O(n \log n + k)$ time.

(**Hint:** *Use a sweep-line algorithm and maintain a segment tree.*)

Problem 2: Show that the space requirement of the 2-dimensional orthogonal range searching can be improved to $O(n)$, provided we allow query time to be $O(n^\epsilon)$, for any arbitrarily small constant $\epsilon > 0$. Of course, the constant of proportionality depends on ϵ. What is the preprocessing time? (**Hint:** Store the secondary structures only at certain levels of the primary tree.)

Problem 3: A circular disk of radius r centered at point $c \in \mathbb{R}^2$ is the set $D = \{x \mid \|x - c\| \leq r\}$. Let $\mathcal{D} = \{D_1, \ldots, D_n\}$ be a set of n circular disks in the plane. Let U be the union of the disks in $\mathcal{D}$. Show that U has $O(n)$ vertices. Describe an algorithm for computing U.

(**Hint:** *Show that each D_i can be mapped to a halfspace H_i in $\mathbb{R}^3$ so that each point in U maps to $\bigcap_i H_i$.*)

Problem 4: The *farthest neighbor Voronoi diagram* of a set S of points in $\mathbb{R}^d$, denoted by $\mathrm{Vor}_f(S)$, is the decomposition of $\mathbb{R}^d$ into maximal connected regions so that the farthest point of S from any point within each region (under the Euclidean metric) is the same.

(i) Show that $\mathrm{Vor}_f(S)$ in the plane is a tree.

(ii) What is the complexity of $\mathrm{Vor}_f(S)$ in $\mathbb{R}^d$?