Midterm out tuesday.
Collaborations.

Shortest Paths

classical shortest paths.

- dijkstra's algorithm
- floyd's algorithm. similarity to matrix multiplication

Matrices

- length 2 paths by squaring
- matrix multiplication. strassen.
- shortest paths by "funny multiplication."
  - huge integer implementation
  - base-$(n+1)$ integers

Boolean matrix multiplication

- easy.
- gives objects at distance 2.
- gives $nMM(n)$ algorithm for problem
- what about recursive?
- well can get to within 2: let T_k be boolean "distance less than or equal to 2^k. Squaring gives T_{k+1}.
- $O(\log n)$ squares for unit length
- what about exact?

Seidel's distance algorithm for unit lengths.

- log-size integers:
  - parities suffice:
    - square G to get adjacency A', distance D'
      - if D_{ij} even then $D_{ij} = 2D'_{ij}$
      - if D_{ij} odd then $D_{ij} = 2D'_{ij} - 1$
  - For neighbors i, k,
    - $D_{ij} - 1 \leq D_{kj} \leq D_{ij} + 1$

    * exists k, $D_{kj} = D_{ij} - 1$
  - Parities
    * If D_{ij} even, then $D'_{kj} \geq D'_{ij}$ for every neighbor k
    * If D_{ij} odd, then $D'_{kj} \leq D'_{ij}$ for every neighbor k, and strict for at least one
  - Add
    * D_{ij} even iff $S_{ij} = \sum_k D'_{kj} \geq D_{ij} d(i)$
    * D_{ij} odd iff $\sum_k D'_{kj} < D_{ij} d(i)$
    * How determine? find $S = AD'$

To find paths: Witness product.

- easy case: unique witness
  - multiply column c by c.
  - read off witness identity
- reduction to easy case:
  - Suppose r columns have witness
  - Suppose choose each with prob. p
  - Prob. exactly 1 witness: $rp(1-p)^{r-1} \approx 1/e$
  - Try all values of r
  - Wait, too many.
- Approx
  - Suppose $p = 2/r$
  - Then prob. exactly 1 is $\approx 2/e^2$
  - So anything in range $1/r \ldots 1/2r$ will do.
  - So try p all powers of 2.
  - suppose $2^k \leq r \leq 2^{k+1}$
  - choose each column with probability 2^{-k}.
  - prob. exactly one witness: $r \cdot 2^{-k}(1 - 2^{-k})^{r-1} \geq (1/2)(1/e^2)$
  - so try $\log n$ distinct powers of 2, each $O(\log n)$ times
- Mod 3:
  - Recall some neighbor distance down by one
  - so compute distances mod 3.
  - suppose $D_{ij} = 1 \bmod 3$
  - then look for k neighbor of i such that $D_{kj} = 0 \bmod 3$
  - let $D^{(s)}_{ij} = 1$ iff $D_{ij} = s \bmod 3$
  - than $AD^{(s)}$ has $ij = 1$ iff a neighbor k of i has $D^{(s)}_{kj}$
  - so, witness matrix mul!

Minimum Cut

- deterministic algorithms
- Min-cut implementation
- data structure for contractions
- alternative view—permutations.
- deterministic leaf algo
- recursion:

$$
\begin{aligned}
p_{k+1} &= p_k - \frac{1}{4}p_k^2 \\
q_k &= 4/p_k + 1 \\
q_{k+1} &= q_k + 1 + 1/q_k
\end{aligned}
$$

- cut counting
- Reliability
- Sampling