Homework 1

§1 Show that $\Sigma_2^p = \mathbf{NP}^{\mathrm{SAT}}$.

§2 Show that $\mathbf{SPACE}(n) \neq \mathbf{NP}$. (Hint: Use padding, mentioned in the notes for Lecture 1.)

§3 Can you give a definition of **NEXPTIME** in terms of certificates as we did for **NP**? If not, report your best attempt.

§4 Say that a class C_1 is *superior to* a class C_2 if there is a machine M_1 in class C_1 such that for every machine M_2 in class C_2 and every large enough n, there is an input of size between n and n^2 on which M_1 and M_2 answer differently.

(a) Is $\mathbf{DTIME}(n^{1.1})$ superior to $\mathbf{DTIME}(n)$?

(b) Is $\mathbf{NTIME}(n^{1.1})$ superior to $\mathbf{NTIME}(n)$?

§5 Suppose we define the *logspace hierarchy* in analogy with the polynomial hierarchy using logspace machines that can use alternation. Does this hierarchy collapse by Immerman's theorem ($\mathbf{NL} = \mathbf{coNL}$)?