Exploration & Representation of Data with Geometric Wavelets

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Problem: Feature Discovery

Terms:

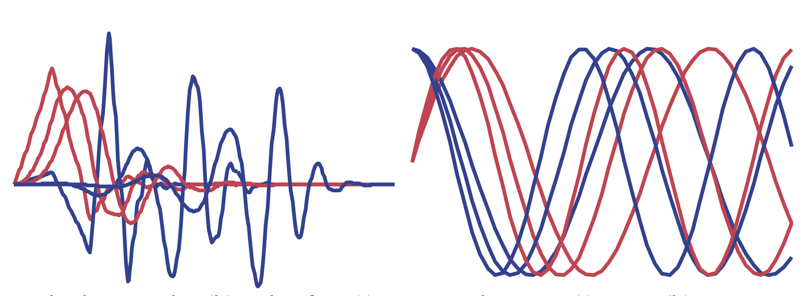
Dictionaries: Sets of characteristic functions or features. We use superpositions of these to build a "model", or data representation.

Low-dimensional signals/functions: e.g. 1D sounds, 2D individual images.

High-dimensional data: Point clouds in D-dimensional space, where each point is one piece of data and D=#pixels/image, #terms/corpus or #samples/spoken vowel.

Challenge:

For low-dimensional signals we have many different **"general-purpose" dictionaries** (Fourier basis, Wavelets, Curvelets) to model our data and do compression, de-noising, sharpening, etc.



Daubechies: wavelets (b), scaling fncts (r) Fourier basis: sine (r), cosine (b)

For high-dimensional data we would like to do the same things, but **we do not have good, tractable models, so we must find features from the data itself.**

Given the data (X) we want to learn a set of features (Φ) and coefficients (α) to build a representation such that

$$X \approx \Phi \cdot \alpha$$

Φ is "good" if α is sparse (lots of zeros or very small values).

Most methods are "black boxes" which have no guarantees, are costly to compute and don't yield interpretable data features.

Our Approach:

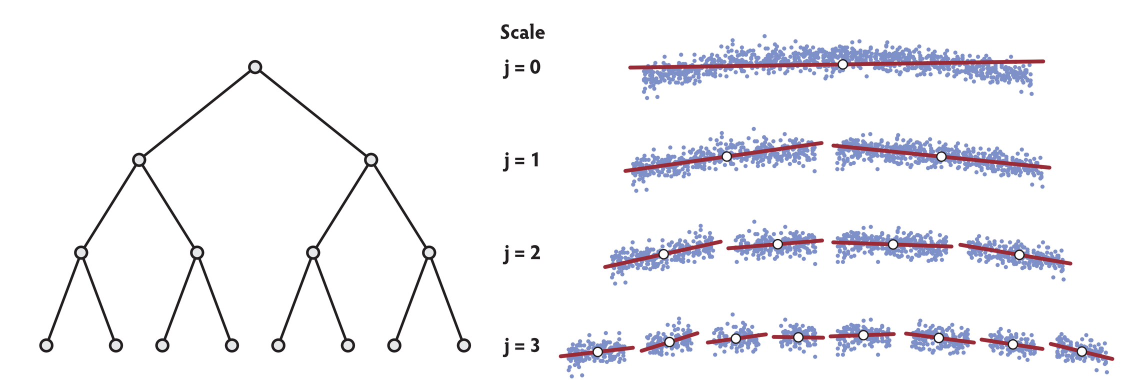
We do not try to solve this problem "in general", but exploit the fact that often **real data has lower-dimensional geometric structure**, such as lying near a **manifold ($\mathcal{M}$)** of dimensionality d<<D.

Geometric Wavelets is a novel construction which discovers features in high-dimensional data under these geometric assumptions.

It is **explicit**, which leads to **interpretable features**, and it comes with **guarantees** (as a function of an approximation error parameter) on **computational cost**, **number of elements in the dictionary** and **sparsity** of the representation. It is **globally non-linear, but piecewise linear**, so it is **fast**, but can **adapt to arbitrary non-linear manifolds**.

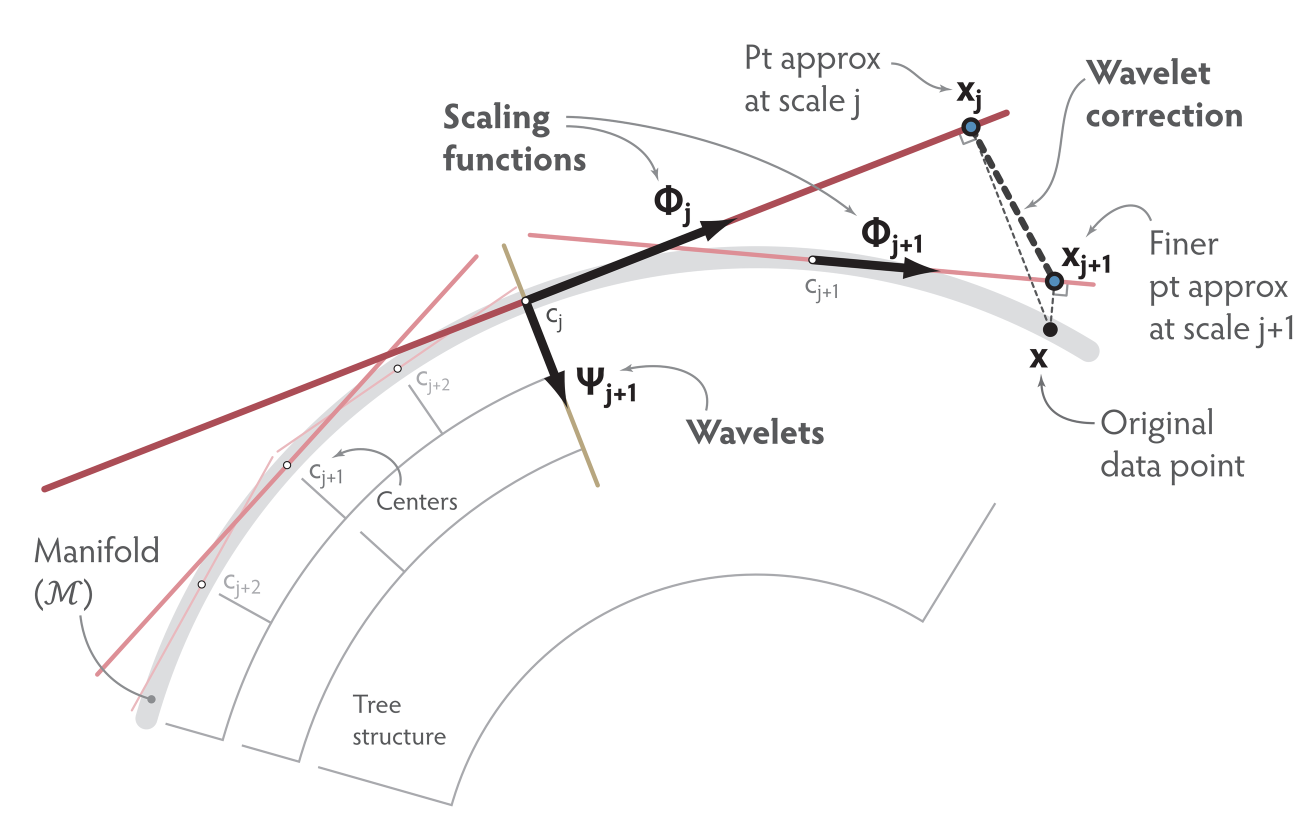
Solution: Geometric Wavelets

1. Cut the data into pieces.
2. Approximate each piece as a low-dimensional plane.
3. Use these approximations as an interpretable representation or feature set (dictionary) for the data that can be used for compression, filtering, outlier detection, etc.

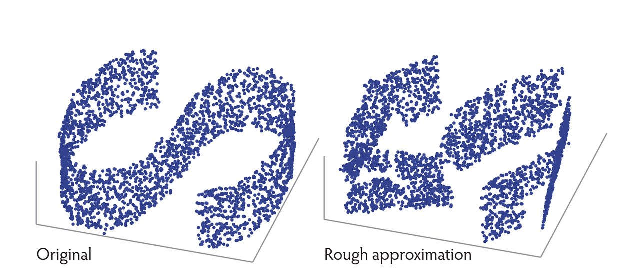


More Details:

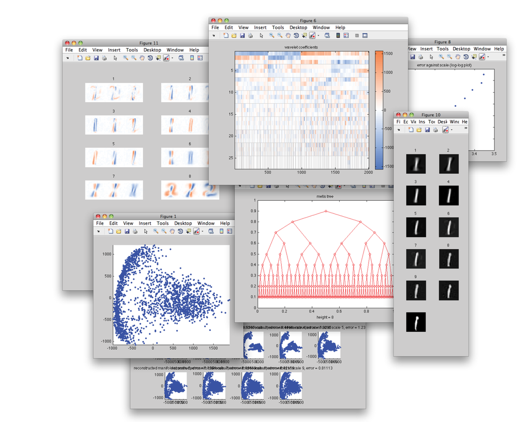
1. Use a similarity measure to create a graph from the data points. Construct a set of multi-scale partitions of $\mathcal{M}$ by using recursive spectral cuts. (METIS uses Eigenfunctions of the Laplacian over the graph – we are also implementing Cover Trees. This step often involves task-specific method variations.)
2. Compute the SVD of the data covariance for each piece. This gives Scaling Functions & $\mathcal{M}_{jk}$ – a manifold approximation at scale j for piece k – a projection onto that local approximate tangent space.
3. Higher-scale planes are small corrections to the previous parent. Efficiently encode the differences between $\mathcal{M}_{j+1} \rightarrow \mathcal{M}_j$ by constructing Wavelet spanning space & "detail" operators analogous to Wavelet theory.



This results in a **multi-scale nonlinear transform** mapping data to a family of pieces of planes which **approximates the original data to any given precision.**

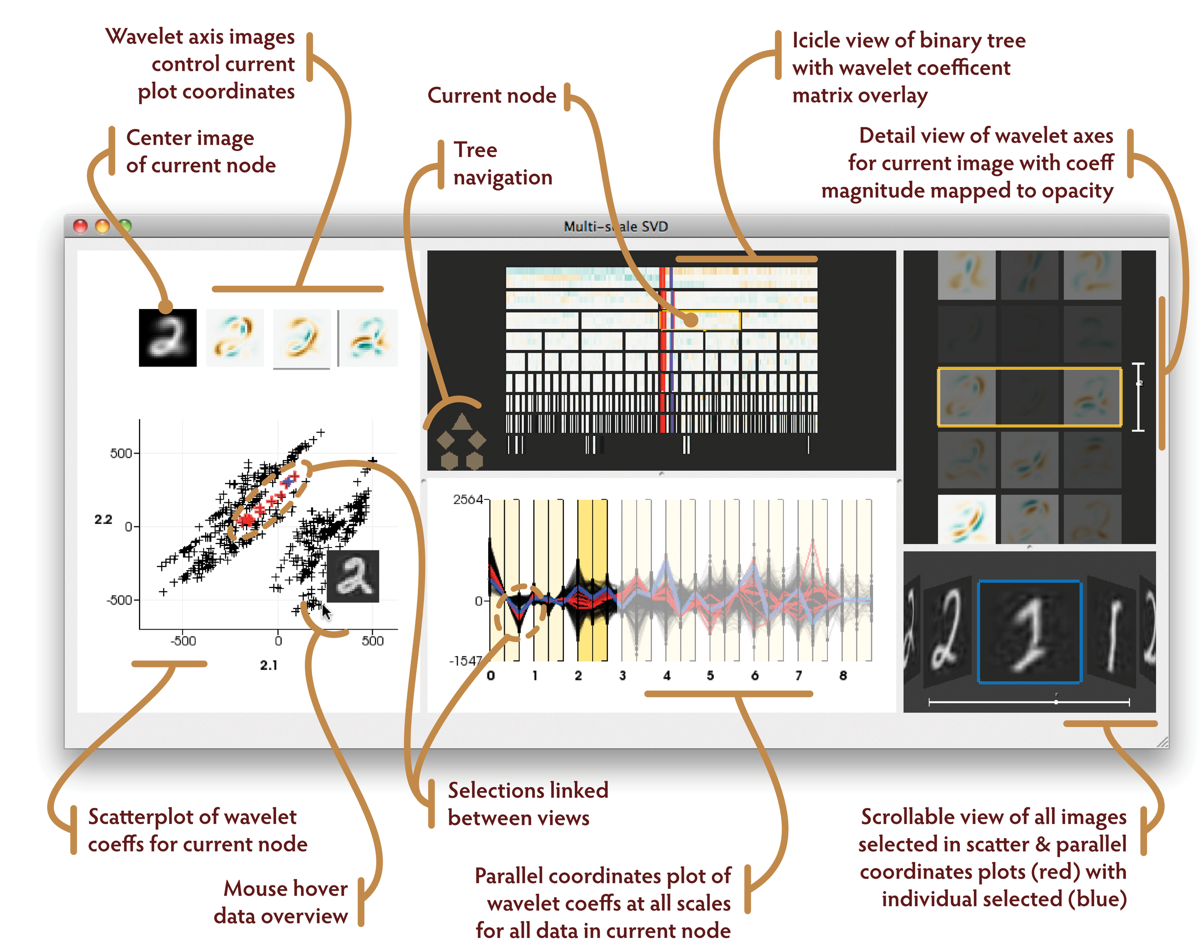


Experience: Interactive GUI

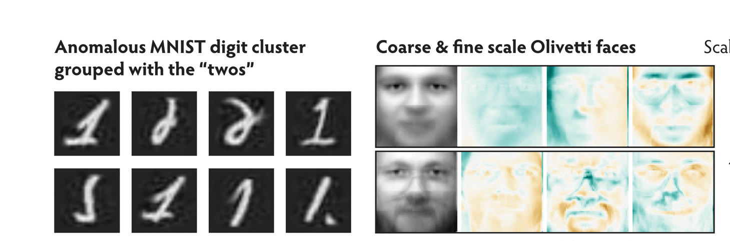


- **Quickly see much more data** so mathematicians can evaluate methods during development.
- Begin **developing a platform** onto which we can build more specialized applications for **new tasks and data types**.
- Help **explain** Geometric Wavelets and **gain intuition** about the representation.

Implemented in Python, using PyQt4 to glue together custom VTK views. Currently the representation is not computed in the GUI – Matlab output is loaded from files.



Observations & Future Directions:



- **Coarser scales contain generalized approximations of the data**, with readily interpretable node centers and wavelet directions.
- **Finer scales reveal anomalous data** through extreme wavelet coefficients or "odd" wavelet axis images.
- Coarser-scale wavelets contain information which could be ignored for classification tasks, but **finer-scale wavelets** encode more specific features which **cluster and characterize individuals**.
- **Developers have changed their ideas** about data encoding after viewing their results in the GUI.
- We are already working on **variable dimensionality**, group definition & labeling for **classification and outlier detection**, views for new data types.