Ellipse Fitting

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Given a set of points, how to find the best ellipse fitting the points? Here we assume that this set of points plausibly belong to a single arc of ellipse. This problem is called *Ellipse Fitting*.

Specifically, let $\mathrm{p}_1, \mathrm{p}_2, \ldots, \mathrm{p}_\mathrm{N}$ be a set of N points, $\mathrm{p_i} = [x_i, y_i]^T$. Let $\mathrm{x} = [x^2, xy, y^2, x, y, 1]^T$, $\mathrm{p} = [x, y]^T$, and

$$f(\mathrm{p}, \mathrm{a}) = \mathrm{x}^T\mathrm{a} = ax^2 + bxy + cy^2 + dx + ey + f = 0$$

the implicit equation of the generic ellipse, characterized by the parameter vector $\mathrm{a} = [a, b, c, d, e, f]^T$. The task is to find the parameter vector, a_0, associated to the ellipse which fits $\mathrm{p}_1, \ldots, \mathrm{p}_\mathrm{N}$ best in the least squares sense, as the solution of

$$min_\mathrm{a} \sum_{i=1}^{N} [D(\mathrm{p_i}, \mathrm{a})]^2$$

where $D(\mathrm{p_i}, \mathrm{a})$ is a suitable distance.

In this talk, I will discuss two possible suitable distances and corresponding solutions to this problem. One is Euclidean distance and the other is algebraic distance.

References

[1] Emanuele Trucco and Alessandro Verri. Introductory Techniques for 3-D Computer Vision. Prentice Hall, 1998. Chapter 5.