Survey of Absolute Orientation Techniques

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June 7th, 2004

Abstract

The Absolute Orientation problem minimizes the mean squared error between two matched points sets under rigid-body transformations. More verbosely, a free point set A is matched to a fixed point set B such that a matching function $\mu : A \to B$ exists. An error function

$$E = \sum_{\mathbf{a} \in A} ||\mu(\mathbf{a}) - R(\mathbf{a}) + \mathbf{t}||^2$$

is minimized over all rotations $R(\cdot)$ and translations $\mathbf{t}$. It turns out that the optimal translation is easy to solve and can be decoupled from the problem of finding the optimal rotation, which is a little more involved.

Iterative solutions for this problem date back to the 1950s and 1960s is photogrammetry and psychology literature. In the 1980s and early 1990s, four distinct closed form solutions were discovered, some several times independently. Faugeras and Hebert [3] represent the rotation as a unit quaternion and discovered the first solution in 1983. Horn [4] rediscovered this technique around 1985 and published in 1987 in the most referenced solution to this problem. An alternate solution which represents the rotation as a rotation matrix and uses a singular value decomposition to find the optimal rotation was discovered by Arun *et. al.* [1] in 1986, although a similar computation for computing the angle between subspaces was in Bjorck and Golub [2]. Also representing rotation as a rotation matrix but solving the optimal rotation with the eigenvalue decomposition was discovered independently by Shwarz and Sharir [6] in 1987 and Horn *et. al.* [5] in 1988. A set of degenerate cases was corrected by Umeyama [7] in 1991. A fourth and more obscure solution technique represents rotation and

translations together as dual number quaternions and was discovered by Walker *et. al.* [8].

This survey will attempt to present all four techniques and give proofs (or sketches of proof and intuition) for correctness.

References

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