Overlays of Minimization Diagrams

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If $F = \{f_i\}_{i=1}^n$ is a collection of n functions $f_i : \mathbb{R}^d \to \mathbb{R}$, the *lower envelope* $\varepsilon_F(x) : \mathbb{R}^d \to \mathbb{R}$ is defined as

$$\varepsilon_F(x) = \min_{f_i \in F} f_i(x).$$

The *minimization diagram* $\mathcal{M}_F$ of F is the orthogonal projection of the graph of ε_F onto $\mathbb{R}^d$. $\mathcal{M}_F$ is a collection of maximal connected open faces of dimensions 0 to d such that for each open face σ, ε_F restricted to σ is attained by a fixed subset of F.

The *overlay* $\mathcal{Q}(F, G)$ of two minimization diagrams $\mathcal{M}_F$ and $\mathcal{M}_G$ is a collection of maximal connected open faces such that for each open face σ, ε_F and ε_G restricted to σ is attained by a fixed subset of F and G. The *complexity* of the overlay is the number of such open faces.

I will present the following result by Koltun and Sharir [1]. If the graphs of $F = \{f_i\}_{i=1}^n$ and $G = \{g_i\}_{i=1}^n$ are collections of hyperplanes, then the complexity of $\mathcal{Q}(F, G)$ is $\Theta(n^{\lfloor d/2 \rfloor + 1})$.

References

[1] Vladlen Koltun and Micha Sharir. On overlays and minimization diagrams. In *SCG '06: Proceedings of the twenty-second annual symposium on Computational geometry*, pages 395–401, New York, NY, USA, 2006. ACM Press.