

Rotation Distance and Triangulations

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Imagine two binary trees T_1 and T_2 with n nodes. The *rotation distance* between T_1 and T_2 is the minimum number of rotations needed to convert one tree into another. What is the maximum rotation distance between any two binary trees with n nodes? For $n \geq 11$, $2n - 6$ rotations are sufficient and this bound is tight [STT98]. The bound itself isn't as interesting as the method the authors used to prove it. They show an isomorphism between rotations on binary trees and diagonal flips of triangulated convex polygons. I will present the upper bound.

You don't need to know much about geometry to understand this proof. It's simple, elegant, and short.

References

- [STT98] Daniel D. Sleator, Robert Tarjan, and William P. Thurston. Rotation distance, triangulations, and hyperbolic geometry. *Journal of the American Mathematical Society*, 1(3):647–681, July 1998.