Representing Motions in $\mathbb{SE}(3)$

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I plan to present some interesting ways to represent motion in $\mathbb{SE}(3)$, the space of all rigid body transformations (rotations and translations). It can in fact be easily be decomposed into a translation and a rotation; however, I want to describe an interesting representation known as dual number quaternions which represents the rigid motion as a helical motion. This rarely-used representation dates to W. K. Clifford in 1873 and were studied extensively by E. Study around 1900. However, recently applications have been found in computer vision [2], geometric modeling [1], and robotics.

It will be necessary to quickly review other more common forms of rotation (rotation matrices, quaternions) which along with a translation can be shown equivalent to dual number quaternions. A simple algebraic transformation exists. However, some very cool techniques and analysis are made intuitive through the representation of rigid motion as dual number quaternions. As time permits, I will demonstrate their power and elegance through a couple of examples.

References

[1] Helmut Pottmann, Qi-Xing Huang, Yong-Liang Yang, and Shi-Min Hu. Geometry and convergence analysis of algorithms for registration of 3D shapes. Technical Report 117, Geometry Preprint Series, TU Wien, June 2004.

[2] Michael W. Walker, Lejun Shao, and Richard A. Volz. Estimating 3-D Location Parameters Using Dual Number Quaternions. *CVGIP: Image Understanding*, 54(3):358–367, November 1991.