

# The Poyla-Redfield Method

Jeff M. Phillips

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## Abstract

Consider the problem of creating a space cage match against Aardvarks, Buffalos, and Chimpanzees. The cage is a cube and an animal is placed in each corner, but since there is no gravity, the orientation of the cube is of no importance, but the placement of all types of animals adjacent to each other is relevant because of the politics within the animal kingdom. We want to determine the number of possible Aardvark-Buffalo-Chimpanzee space cage matches possible. Call this the ABC-SCM problem.

The Poyla-Redfield Method solves the ABC-SCM problem and many other related problems efficiently by counting the symmetries instead of enumerating all possible configurations and then comparing the symmetries. Although at first glance the method is a little confusing, it is extremely simple and elegant.

The original idea for the method dates back to W. Burnside in 1911. J. H. Redfield rediscovered it in 1927 to analyze chemical compounds. But the use of this method did not become immensely popular until it was again rediscovered by George Poyla (a father of combinatorics) in 1937. In what is described by R. C. Read in 1987 as "... a remarkable theorem in a remarkable paper, and a landmark in the history of combinatorial analysis." Poyla formalized the method and abstracted it to the enumeration of chemical compounds, the enumeration of rooted trees in graph theory, and many other more practical applications such as the ABC-SCM problem. In fact, the field of enumerative graph theory grew out of this theory.

I will present the Poyla-Redfield method through a series of examples of counting problems of increasing difficulty along the way developing the infrastructure for the full use of the Poyla-Redfield method.