The Van der Corput Sequence and Other Low Discrepancy Sequences

Jeff M. Phillips

Given a point set P, a domain D, and a family of ranges $\mathcal{R}$, P is said to be an ε-approximation of D if

$$\max_{R \in \mathcal{R}} \left| \frac{|R \cap P|}{|P|} - \frac{|R \cap D|}{|D|} \right| \leq \varepsilon,$$

where the $|\cdot|$ operator either represents the cardinality of a finite set or the natural measure of some continuous set. If $\mathcal{R}$ has bounded VC-dimension and a set P of size $O\left((\frac{1}{\varepsilon})^2 \log \frac{1}{\varepsilon\delta}\right)$ is selected uniformly at random from D then P is an ε-approximation with probability $1 - \delta$. However, for certain families of ranges $\mathcal{R}$, such as axis-parallel rectangles and constant-size polygons, ε-approximations can be deterministically constructed with size $O(\frac{1}{\varepsilon} \log \frac{1}{\varepsilon})$.

In this talk I will focus on the case where D is an axis-parallel unit square (or hypercube) and $\mathcal{R}$, the family of ranges, is axis-parallel rectangles. For the 2-d variation of this case, the Van der Corput sequence provides a deterministic construction for an ε-approximation with size $O(\frac{1}{\varepsilon} \log \frac{1}{\varepsilon})$. I will prove this result by proving that the Van der Corput sequence has logarithmic discrepancy for $\mathcal{R}$. As time provides, I will generalize this result to higher dimensions and describe other related results.