CPS 100 **Exam 2 Solutions** **Spring 1995**

Dr. Rodger

1. (3 pts) A virtual function in C++ is bound dynamically or statically?

   dynamic

2. (3 pts) When does one use a class template in C++?

   Templates are used to represent classes of different data types.

3. (3 pts) Algorithm 1 takes $N^2 + 1$ steps. Algorithm 2 takes $5N + 10$ steps. For what values of N, an integer > 0, is Algorithm 1 faster than Algorithm 2.

   0 < N < 7

4. (6 pts) Consider Quicksort and Mergesort.

   (a) Which one does most of its work before its recursive calls? **Quicksort**

   (b) Explain why one of these is more reliable (in running time) than the other?
   Mergesort is more reliable because its worst case running time is $O(n \log n)$. Quicksort's usually runs in $O(n \log n)$ time, but its worst case running time is $O(n^2)$.

5. (10 pts) Consider the operation FINDMIN(D) where D is a data structure containing N items and FINDMIN returns the value of the minimum key in the data structure.

   What is the **worst case** running time of this operation on the following data structures?

Data Structure D	running time of FINDMIN
array - not in sorted order	$O(n)$
array - in sorted order	$O(1)$
binary search tree	$O(n)$
2-3 tree	$O(\log n)$
mintree	$O(1)$

6. (9 pts) What is the worst case running time (big-O) of the following?

   (a)
```
int count = 0;
for (int i=1; i <= N; i+=4) {
      count = count + 1;
}
```
   $O(N)$

(b)
```
int count = 0;
int i = 1;
while (i<M) {
   for (int j=i; j<N; j++) {
      count = count + 1;
   }
   i = i + 1;
}
```
$O(M + N^2)$

(c)
```
int count = 0;
int i,j,k;
for (i=1; i <= M; i++) {
   for (j=1; j<=N; j=j*2) {
      for (k=1; k<=P; k++) {
         count = count + 2;
      }
   }
}
```
$O(MP \log N)$

7. (10 pts) A **trinary** tree is a tree in which each node has at most three children. A **strictly trinary** tree is a trinary tree in which each nonleaf node has **exactly** three children. Prove by induction that a strictly trinary tree with n leaves has $(3n - 1)/2$ nodes. Show the basis, induction hypothesis (IH) and induction step (IS).

   - Proof: Basis:(N=1) ((3*1-1)/2 = 1)

     I.H.: Assume a strictly trinary tree with k leaves has exactly $(3k - 1)/2$ nodes for $k < N$.

     I.S.: Show a strictly trinary tree with N leaves has exactly $(3N - 1)/2$ nodes.

     Given a strictly trinary tree with N leaves. Remove 3 leaves that have the same parent. The tree is still a strictly trinary tree with N-2 leaves (removed 3 leaves, but the parent is now a leaf).

     By I.H. this tree has $(3 * (N - 2) - 1)/2$ nodes = (3N-7)/2 nodes.

     If you add the three leaves back where they belonged, then the original tree has (3N-7)/2+3 = (3N-1)/2 nodes.

8. (10 pts) Solve the following recurrence relation. T(n) = O(?). Show all steps.

   $T(1) = 1$

   $T(n) = 4\ T(n/2) + n^2$

   - $T(n) = 4\ T(n/2) + n^2$

     $= 4\ [4\ T(n/4) + (n/2)^2] + n^2$

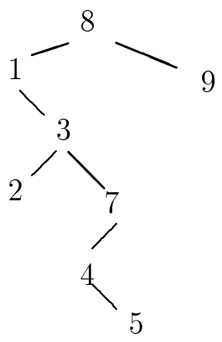
     $= 4^2\ T(n/4) + 2n^2$

$= 4^2$ [4 T(n/8) + $(n/4)^2$] + $2n^2$
$= 4^3$ T(n/8) + $3n^2$
$= 4^k$ T($n/2^k$) + kn^2
(Let $n/2^k$=1, $\rightarrow n = 2^k$ and $k = \log n$ and $n^2 = 4^k$)
$= n^2$ T(1) + $\log n * n^2$
$= O(n^2 \log n)$

9. (10 pts) Insert the following integers (in the order given) into an empty binary search tree.

   8,1,3,7,9,4,2,5

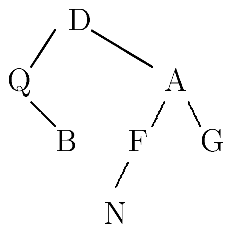
   The resulting tree is:



10. (10 pts) Given the following traversals of a binary tree, draw the tree.

    PREORDER: DQBAFNG

    INORDER: QBDNFAG



11. (12 pts) A node in a binary tree is described below. Write a function (in C++) called **height** that returns the height of a binary tree (an integer). (For example, the height of a binary tree with just one node is 1).

```
struct node {
    int data;
    node * lc;    // left child
    node * rc;    // right child
};

 int height(node * t) {
     if (t== NULL)
        return 0;
     else {
        int left = height(t->lc);
        int right = height(t->rc);
        if (left > right)
           return left+1;
        else
           return right+1;
        }
 }
```

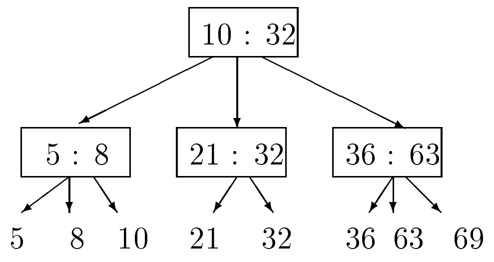
If there are n nodes in the binary tree, what is the running time of your function?

$O(n)$

12. (14 pts) Write a function (in C++) to print out a 2-3 tree sideways in the following manner: starting with the rightmost (largest) leaf, printing one node per line, and using indentation for different levels. (Hint: use a counter to determine which level of the tree you are on, and use that for indenting). For the tree:

A node for a 23-tree is defined as:

```
struct node23 {
   int type;       // 1 = leaf node, 2 = nonleaf node
   // following fields used for nonleaf nodes
   int sleft;      // largest value in left subtree
   int smid;       // largest value in middle subtree
   node23 * lc;    // pointer to left child
   node23 * mc;    // pointer to middle child
   node23 * rc;    // pointer to right child
   // following field used for leaf nodes
   int data;       // data item
}
```



The output is:

```
                69
                63
        36 : 63
                36
                32
        21 : 32
                21
10 : 32
                10
                8
        5 : 8
                5
```

```
void print(node23 * t, int level) {
   if (t==NULL)
      return;
   if (t->type == 1) {
      indent(level);
      cout << t->data << endl;
   }
   else {
      print(t->rc, level+1);
      print(t->mc, level+1);
      indent(level);
      cout << t->sleft << ":" << t->smid << endl;
      print(t->lc, level+1);
   }
}

void indent(int count) {
   int j;
   for (j=1; j<count; j++)
      cout << "\t";          // print a tab
}
```