Data Structures for Moving Objects

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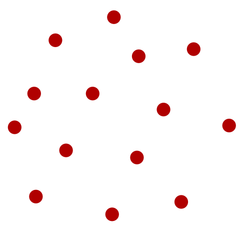
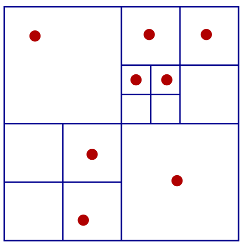
Geometric Data Structures



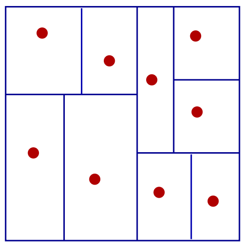
S: Set of geometric objects
Points, segments, polygons

☆ Ask several queries on S

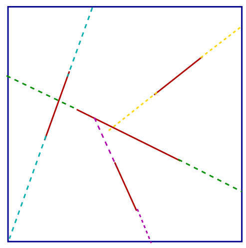
- Range searching
- Nearest-neighbor searching



Quad Tree



kd–Tree



BSP

Dynamic Data Structures

- ☆ Allow insertion/deletion of objects at discrete time instances
- ☆ Several dynamization techniques (Bentley-Saxe; Overmars)
- ☆ Amortized vs worst-case update time (Overmars)
- ☆ Desirable to update the data structure in $O(\log n)$ time

Moving Objects: Applications

- ☆ Traffic management
  - Location based services
  - Emergancy services
  - Air traffic control
- ☆ Digital battlefields
- ☆ Animation
- ☆ Deformable objects
- ☆ Adhoc networks

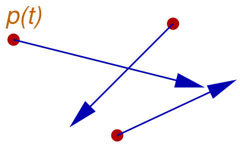
Need data structures for storing, analyzing, querying moving objects.

Dynamic data structures not suitable for handling moving objects!

Modeling Motion

$p(t) = (x(t), y(t))$: Position of p at time t.

- $x(\cdot), y(\cdot)$: Polynomials
- *Degree* of motion: max degree of $x(\cdot), y(\cdot)$.
- *Linear motion:* Degree of motion is 1
  $p(t) = \mathbf{a}t + \mathbf{b}, \quad \mathbf{a}, \mathbf{b} \in \mathbb{R}^2$



☆ Mostly assume motion to be linear

☆ Trajectory of points can change

☆ Trajectory can be piecewise linear

Issues:

☆ Sampled motion

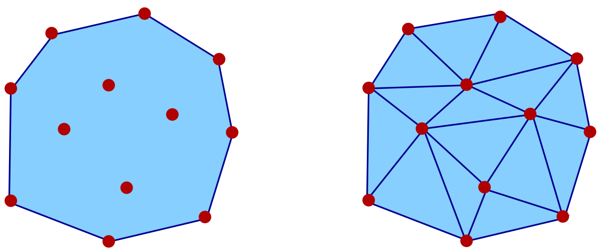
☆ Hierarchical motion

☆ Uncertainty

Previous Work

☆ Off-line setting: Linear motion, known in advance

☆ Bound the number of *combinatorial* changes in various geometric structures as points move
E.g. Convex hull, closest pair, Delaunay triangulation



CH(S) DT(S)

☆ Number of changes in $CH(S)$: $\Theta(n^2)$

☆ Number of changes in $DT(S)$: $O(n^2), \Omega(n^2)$
(A.-Sharir, 2000)

Previous Work

Similar problems arise in combinatrial optimization

- ☆ *Senstivity analysis* of optimization problems:
- ☆ *Kinetic minimum spanning tree*:
  - $G = (V, E)$, $w(e) = a \cdot t + b$
  - How many combinatorially different MST's are there?
  - \# different MSTs = $O(mn^{1/3})$ (Gusfield, 1979; Dey, 1998)
- ☆ Similar questions for max flow, shortest paths, ...

Time Sampling Approach

- ☆ Choose fixed time step
- ☆ Update the positions of moving objects at each time stamp
- ☆ Update the data structure with the new positions of objects
- ☆ Use temporal coherence to expedite the updates

Works well when

- ☆ Few objects are moving
- ☆ All of them are moving with roughly the same speed

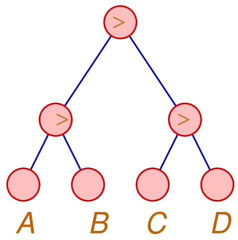
Issues:

- ☆ Expensive if many objects are moving
- ☆ How to choose the time step

Kinetic Data Structures

Exploit combinatorial structure of the algorithm
(Basch, Guibas, Hershberger, 1998)

- ☆ Maintain a set of *certificates*
- ☆ Certificates provide a proof of the correctness of the structure
- ☆ Determine when a certificate fails; an *event*
  - Event times are roots of certain polynomials
- ☆ Update the structure and compute new crtificates
- ☆ A global priority queue stores the events.

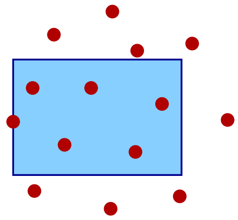


Computing max

Kinetic Data Structures

- ☆ Performance of a KDS depends on
  - Number of events
  - Number of certificates
  - Time spent at each event
- ☆ Useful for physical simulation
- ☆ Not always suitable for data structures
  - Need to predict future behavior based on current trajectories
  - Why update the data structures if no queries are being asked
  - Global priority queue is not always feasible

Range Searching



S: Set of points

Preprocess S into a data structure
Report all points of S lying inside a query rectangle

Data Structure	Space	Query
Range tree	$n\frac{\log n}{\log\log n}$	$O(\log n + k)$
External Range tree	$\frac{n}{B}\frac{\log_B n}{\log_B \log n}$	$O(\log_B n + k/B)$
kdB-tree	$O(n/B)$	$O(\sqrt{n/B} + k/B)$

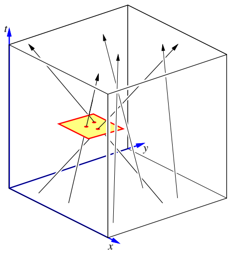
Kinetic Range Searching

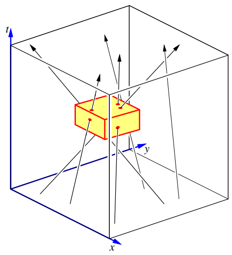
S: Set of points, each moving with fixed velocity in the plane

Preprocess S into a data structure:

Q1 Given a rectangle R and a time value t,
report all points of $S(t) \cap R$

Q2 Given a rectangle R and time values t_1, t_2,
report all points that pass through R during the time interval $[t_1, t_2]$.





Kinetic Range Searching

(A., Arge, Erickson, 2001)

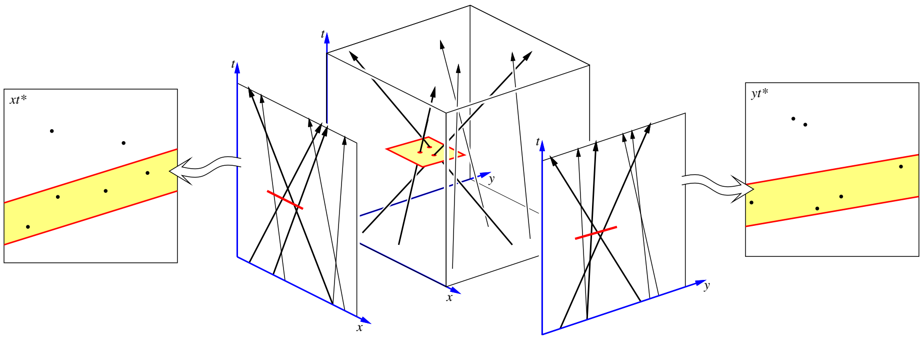
☆ Partition-tree based approach

- $O(n)$ space, $\approx \sqrt{n} + k$ query time
- $O(\log^2 n)$ insertion/deletion/trajectory-change
- Time oblivious scheme

☆ Kinetic range trees

- $O(n \log n / \log\log n)$ space, $O(\log n + k)$ query
- Events: x- or y-coordinates of two points become equal
- $\Theta(n^2)$ events, each requiring $O(\log^2 n)$ time
- Tradeoff between # events and query time
- Queries have to arrive in a chronological order

Partition Tree Based Approach



☆ Trajectory of a point p_i is a line ℓ_i in $\mathbb{R}^3$

☆ $p_i(t) \in R \Longleftrightarrow \ell_i$ intersects (R, t)

☆ ℓ_x, ℓ_y: Projection of ℓ onto the xt- & yt-planes

☆ ℓ intersects $(R, t) \Leftrightarrow \ell_x$ intersects (R_x, t) & ℓ_y intersects (R_y, t)

☆ Use duality and partition trees

Kinetic Range Searching

Time Responsive Approach:

- ☆ Near-future queries are more critical than far-future queries
- ☆ Fast query time for near-future queries
- ☆ Measure *future* by the number of events occurred
  - Measures how much the point configuration has changed

(A., Arge, Erickson, 2000)

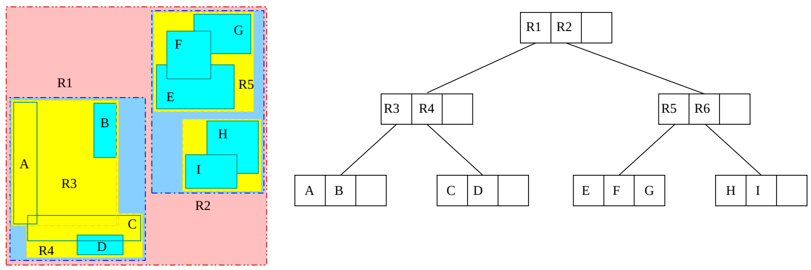
- ☆ Combine partition trees and kinetic range trees
- ☆ Space $O(n \log n / \log \log n)$, # events $O(n^2)$
- ☆ Prove bounds on the query time if trajectories are uniformly distributed

(A., Arge, Varenhold, 2001)

- ☆ Similar bounds but without any assumption on trajectories

R-trees

- ☆ Bounding box hierarchy, B-tree
- ☆ Each node v is associated with a subset S_v of points and the smallest rectangle R_v containing S_v
- ☆ Partition S_v into B clusters, each associated with a child of v
- ☆ Several heuristics are proposed for partitioning S_v into B clusters



Kinetic R-trees

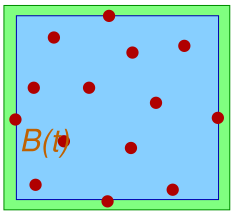
☆ Maintain the smallest box enclosing the set of moving points

- Use a KDS to maintain the box
- The combinatorial structure can change $\Omega(n)$ times
- Maintain an approximation of the smallest enclosing box

☆ Maintaining the clustering kinetically

- Extend the known heuristics
- No theoretical nontrivial results known on kinetic clustering

Smallest Enclosing Box



☆ $B(t)$: Smallest box enclosing $S(t)$

☆ $B^{\varepsilon}(t)$: Outer ε-approx of $B(t)$

$$B(t) \subseteq B^{\varepsilon}(t) \subseteq (1+\varepsilon)B(t)$$

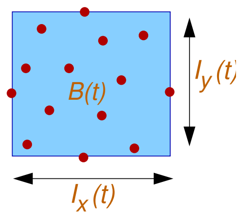
Can one maintain $B^{\varepsilon}(t)$ so that its combinatorial structure changes only a few times?

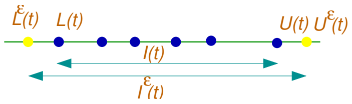
(A., Har-Peled, 2001)

☆ # changes $O(1/\sqrt{\varepsilon})$, total time $O(n \log(1/\varepsilon))$

☆ Supports insertions, deletions, change in trajectory in $O(\log^2 n)$ time

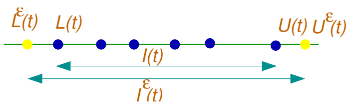
Smallest Enclosing Box

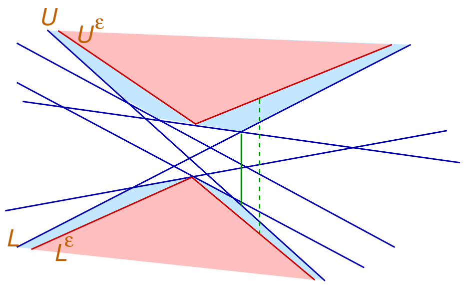




- ☆ Suffices to compute the *extent* in each dimension separately
- ☆ $p_i(t) = a_i t + b_i$
- ☆ $L(t) = \min_i a_i t + b_i$, $U(t) = \max_i a_i t + b_i$
- ☆ $I(t) = U(t) - L(t)$
- ☆ $L(t) - (\varepsilon/2)I(t) \leq L^\varepsilon(t) \leq L(t)$,
  $U(t) \leq U^\varepsilon(t) \leq U(t) + (\varepsilon/2)I(t)$,
- ☆ $I^\varepsilon(t) = U^\varepsilon(t) - L^\varepsilon(t)$

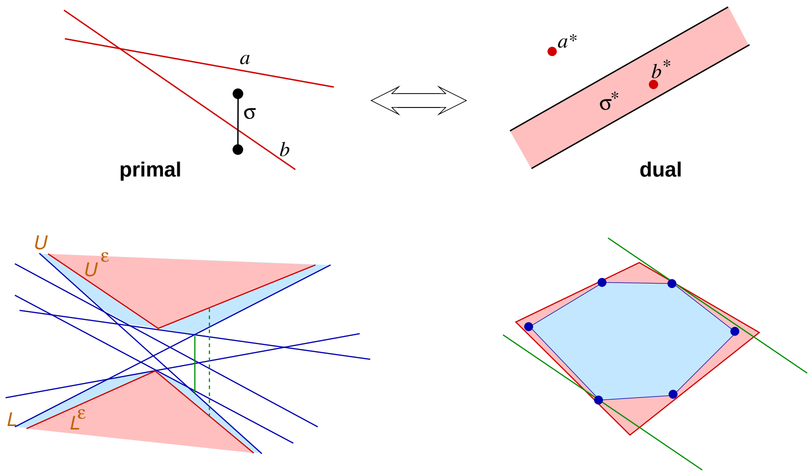
Geometric Interpretation





- ☆ $\ell_i = \bigcup_t (t, p_i(t))$: Trajectory of p_i
- ☆ L: Lower envelope of trajectories; U: Upper envelope
- ☆ L^ε: ε-approximation of the lower envelope
- ☆ U^ε: ε-approximation of the upper envelope

Geometric Interpretation: Duality



Approximate envelope $\Longrightarrow$ Approx convex hull of dual points

Summary of Results

☆ An ε-approximation of convex hull with $O(1/\sqrt{\varepsilon})$ vertices can be computed in $O(n \log(1/\varepsilon))$ time
One can even compute it in an on-line manner

☆ ε-approximation of smallest enclosing box with $O(1/\sqrt{\varepsilon})$ combinatorial changes in $O(n \log(1/\varepsilon))$ time.

☆ Diameter:

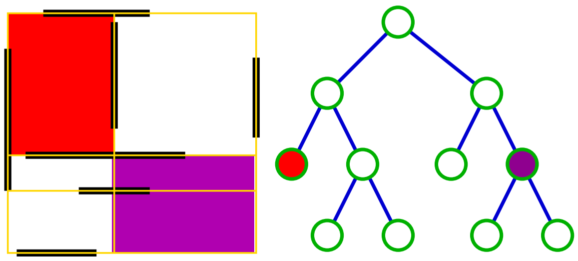
- Maintain the farthest pair of moving points
- Exact diameter: $\Theta(n^2)$ changes (A., Guibas, Hershberger, Veach, 1997)
- ε-approximate diameter: $O(1/\varepsilon)$ changes

☆ Smallest enclosing disk:

- Smallest disk: $O(n^3)$, $\Omega(n^2)$ changes
- ε-approximation: $O(1/\varepsilon^{3/2})$ changes

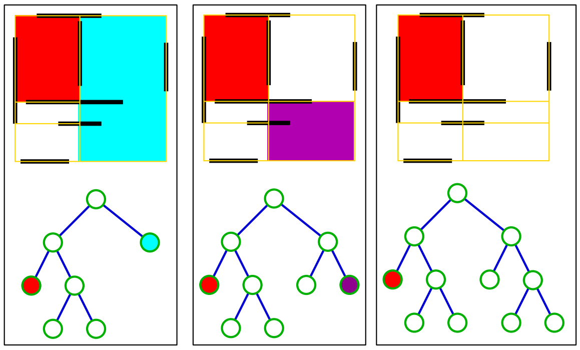
Binary Space Partitions (in $\mathbb{R}^3$)

- Each node v is associated with a convex polytope τ_v.
- If v is a leaf, τ_v is empty; otherwise, τ_v is split by a plane h_v
- h_v and triangles lying in h_v stored at v.



- Leaves induce a partition of the space
- BSP induces a hierarchical partition of $\mathbb{R}^3$.

Binary Space Partitions (in $\mathbb{R}^3$)



Kinetic BSP

$S(t)$: Objects at time t
(segments in 2D, triangles in 3D).

$\mathcal{A}$: An algorithm for constructing a BSP

$B(t)$: BSP for $S(t)$ constructed by $\mathcal{A}$.

Maintain $B(t)$ efficiently.

Commonly used approach:

Discretize time into short intervals and modify the BSP at the end of each interval — *insert/delete objects that have moved*

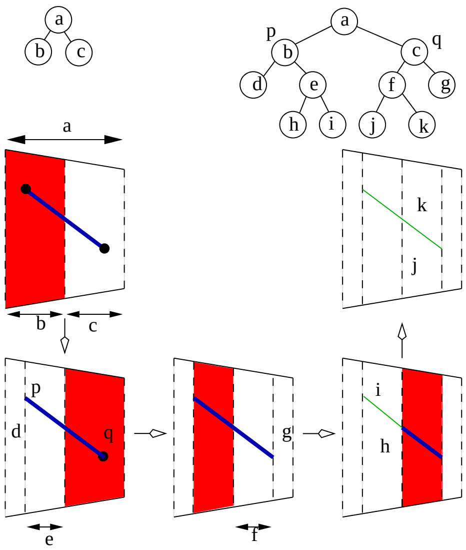
How does one choose the time interval?

☆ *Long interval:* May miss some updates

☆ *Short interval:* Unnecessary computation

BSP for Segments

Randomized incremental algorithm: Add a random segment



BSP: KDS Framework

☆ Maintain the BSP of $S(t)$ constructed by the randomized incremental algorithm

- Permutation remains fixed

☆ Maintain a set of *certificates* to keep track of when the structure of BSP changes.

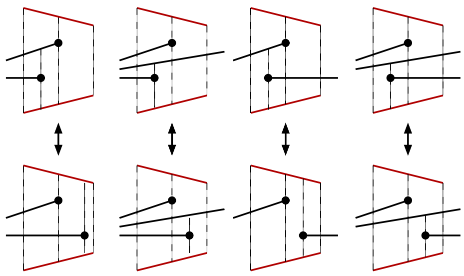
☆ *Critical events:* One of the certificates becomes invalid.

☆ Update the BSP at critical events.

☆ *Assume coin-flips are private.*

☆ At each event:
Expected # certificate changes = $O(1)$
Expected $O(\log n)$ time to update the BSP at a critical event.

Critical Events for Segments



Kinetic BSP

k: # moving objects, each moving along a pseudo-algebraic trajectory

- Disjoint segments
  Expected update time: $O(\log n)$
  Expected # events: $O(nk)$
- Set of intersecting segments
  Expected update time: $O(\log n)$
  Expected # events: $O(nk\lambda_s(n))$
- Set of disjoint triangles in 3D
  Expected update time: $O(\log^2 n)$
  Expected # events: $O(nk\lambda_s(n) \log n)$

Update algorithm does not require trajectories to be pseudo-algebraic.

Concluding Remarks

- Incorporating more realistic motions
- Hierarchical representation of motion
- Tradeoff between updates and queries
- Practical kinetic data structure for nearest-neighbor queries