Due Date: February 25, 2003

Problem 1:

(i) Show that $\leq_m^{\mathsf{L}}$ is transitive.

(ii) Is NP closed under $\leq_m^{\mathsf{P}}$? What about $\leq_T^{\mathsf{P}}$?

Problem 2: Show that if $\mathsf{DTIME}(2^{cn}) \subset \mathsf{NP}$ for some c, then

$$\mathsf{NP} = \mathsf{PSPACE} = \mathsf{EXPTIME}.$$

Problem 3: Given a directed graph G and two vertices u, v, the *graph reachability* problem is to determine whether there exists a path from u to v in G. Show that the graph reachability problem is NL-complete under the $\leq_m^{\mathsf{L}}$-reduction.
(**Hint:** *Show that the configuration graph of a* $\mathsf{LOGSPACE}$ *nondeterministic Turing machine can be constructed by a DTM using only* $O(\log n)$ *space. Recall that one does not take the output size into account.*)

Problem 4: Let $G = (V, E)$ be a directed graph. A subset $D \subseteq V$ is called a *dominating subset* of G if for every $v \notin D$ there is a $u \in D$ with $(u, v) \in E$. Show that the following language is NP-Complete:

$$DS = \{(G, k) \mid G \text{ has a dominating set of size} \leq k\}.$$