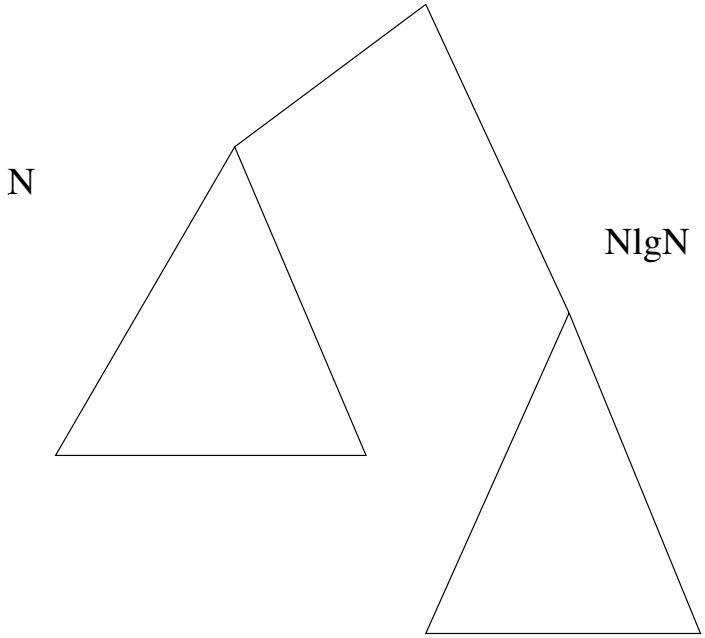
9.1-3 Show that there is no sorting algorithm which sorts at least $(1/2^n) \times n!$ *instances in* $O(n)$ *time.*

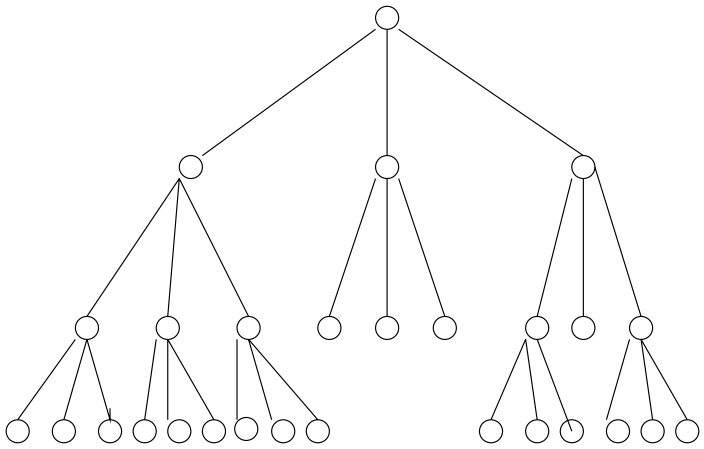
Think of the decision tree which can do thsi. What is the shortest tree with $(1/2^n) \times n!$ leaves?



$$\begin{aligned} h > \lg(n!/2^n) &= \lg(n!) - \lg(2^n) \\ &= \Theta(n \lg n) - n \\ &= \Theta(n \lg n) \end{aligned}$$

Moral: there cannot be too many good cases for any sorting algorithm!

9.1-4 Show that the $\Omega(n \lg n)$ lower bound for sorting still holds with ternary comparisons.



The maximum number of leaves in a tree of height h is 3^h,

$$\lg_3(n!) = \Theta(n \lg n)$$

So it goes for any constant base.