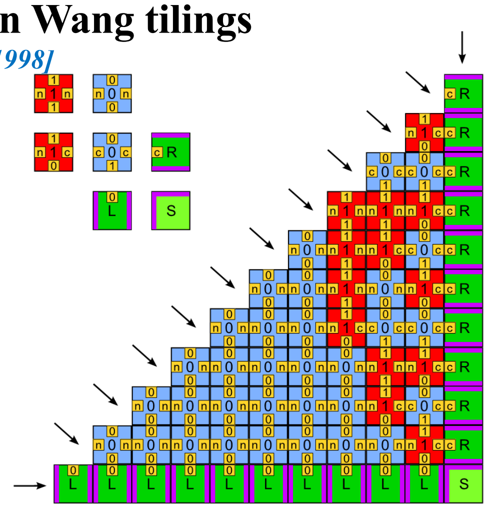
Tile Complexity of Assembly of Length N Arrays and N x N Squares

by John Reif and Harish Chandran

Tile assembly model (TAM)

- **Proposed by Erik Winfree developing on Wang tilings**
  - *[Winfree: Simulations of Computing by Self-Assembly, 1998]*
- **Simple, yet powerful model**
- **Refines Wang tiling**
- **Models crystal growth**
- **Also, Turing-complete**
- **Can be implemented using DNA molecules**



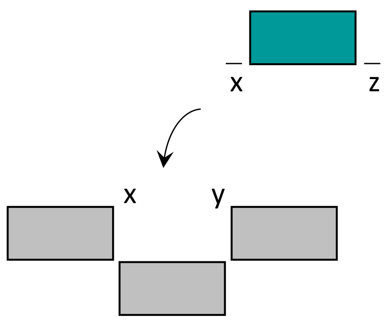
Abstract Tile Assembly Model:

[Rothemund, Winfree, '2000]

Temperature: A positive integer. (Usually 1 or 2)

A set of tile types: Each tile is an oriented rectangle with glues on its corners. Each glue has a non-negative strength (0, 1 or 2).

An initial assembly (seed).



A tile can attach to an assembly iff the combined strength of the "matched glues" is greater or equal than the temperature T.

(Chen)

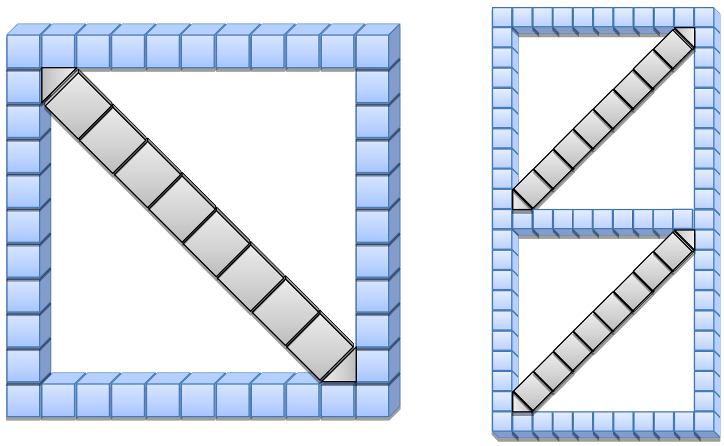
Tile Complexity

- Tile Complexity is the Number of tile types to construct a shape
- Need to minimize the tile complexity
- Implementation constraints
- There are only 4 bases to play with in DNA
- More number of tile types: longer DNA strands
  - High cost and more errors

Linear Deterministic Tile Assemblies of length N Using N tiles

- Linear sequence of N tiles

- Can be used in nanostructures as beam and struts



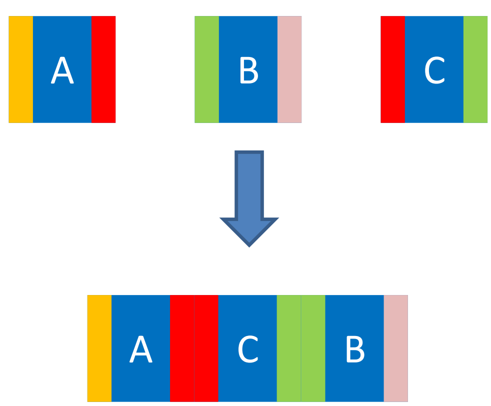
L-TAM Tiling Model for Linear Assemblies

- **L-TAM** is a simplified version of TAM Tiling Model for linear assemblies

- Linear assemblies have no co-operative binding

- Pads on only the East and West side of tiles

- Tiles bind iff their pads match



Output of Deterministic Tiling systems

- **Output of a tile system** is the final shape assembled
- Answer to the instance of problem being solved
- For a system under TAM:
- Exactly one final shape is produced
- One output for an instance of a problem
- Reason: at any given position in a partial assembly, exactly one tile type can attach
- Deterministic constraint of TAM

Tile Complexity of Assembly of Given Size or Shape

Assume TAM model of Tiles

- **Size Problem:**
  - **Given shape with defined size, assemble (with give size) using smallest number of tiles.**
  - **Examples:**
    - **Linear Assembly Problem:**
      - **given length N, assemble a 1 x N rectangle**
    - **Square Assembly Problem:**
      - **given length N, assemble a N x N square**
- **Shape Problem:**
  - **Given shape with defined size, assemble shape (of any size) with smallest number of tiles.**

Results in Deterministic Tiling Assembly

- **Efficiently assembling basic shapes with precisely controlled size and pattern:**
  - Constructing N X N squares with O(log n/log log n) tiles. *[Adleman, Cheng, Goel, Huang,2001]*
  - Perform universal computation by simulating BCA. *[Winfree 2099]*
  - Assemble arbitrary shape by O(Kolmogorov complexity) tiles with scaling. *[Soloveichik, Winfree 2004]*

Results for Deterministic Tiling Complexity

- **Assume TAM model of Tiles (temperature τ)**
- **Deterministic Tile Set:**
  - **require that only one assembly be possible for given set of tiles**

- **Linear Assembly Problem: temperature τ=1**
  - **given length N, uniquely assemble a 1 x N rectangle**
  - **has tile complexity Θ(N)**

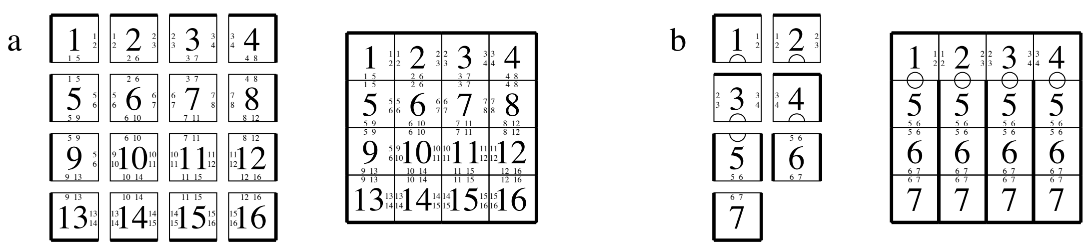
- **Square Assembly Problem:**
  - **given length N, uniquely assemble a N x N square**
  - **Temperature τ=1**
    - **Rothemund & Winfree:**
      - **has tile complexity: $O(N^2)$**
  - **Temperature τ=2**
    - **Rothemund & Winfree: lower bound at least Ω(log(N)/loglog(N)).**
    - **Rothemund & Winfree: upper bound at most O(log(N))**
    - **Adelman: upper bound improved at most O(log(N))/loglog(N)))**
    - **=> has tight bounds on tile complexity: Θ(log(N))/loglog(N)))**

Deterministic Temp τ=1 Tiling Complexity

- **Linear Assembly Problem: temperature τ=1**
  - **given length N, uniquely assemble a 1 x N rectangle**
  - **has tile complexity Θ(N)**

- **Square Assembly Problem:**
  - **given length N, uniquely assemble a N x N square**
  - **Temperature τ=1**
    - **Rothemund & Winfree:**
      - **has exact tile complexity: $\Theta(N^2)$**

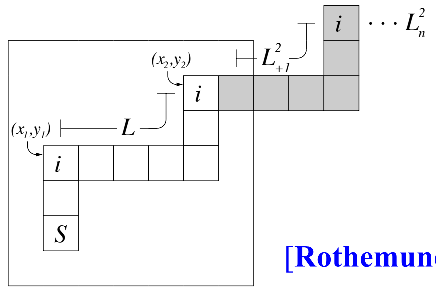
Deterministic Temp τ=1 Square Tiling Complexity



Formation of squares at $\mathcal{T} = 1$. (a) $N^2 = 16$ tiles with unique side labels uniquely produce a terminal 4×4 full square at $\mathcal{T} = 1$. (b) $2N-1 = 7$ tiles uniquely produce a 4×4 square (but this is not a full square since thick sides have strength 0). Except for the sides labeled with a circle, each interacting pair of tiles share a unique side label. This comb-like construction is conjectured to be minimal for $N{\times}N$ squares assembled at $\mathcal{T} = 1$. [Rothemund & Winfree, 2000]

- **Square Assembly Problem:**
  - **given length N, uniquely assemble a N x N square**
  - **Temperature τ=1**
    - **Rothemund & Winfree: Upper Bounds:**
      - **has tile complexity at most $O(N^2)$**

Deterministic Temp τ=1 Square Tiling Complexity



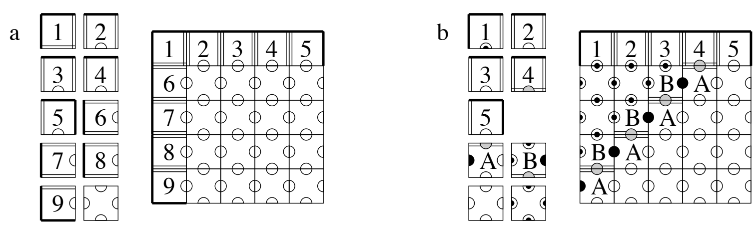
[Rothemund & Winfree, 2000]

No $\mathcal{T}=1$ tile system with fewer than N^2 tiles can uniquely produce an $N \times N$ square. A full $N \times N$ square with fewer than N^2 tiles must have some tile i present at two sites. Consider the assembly R (the white tiles) which includes an assembly L (bounded by the tiles i), the seed tile S, and a tile that connects the seed tile to L. R can be extended indefinitely with the addition of translated segments of L (*e.g.* L^2_{+1} shown in gray).

Square Assembly Problem:

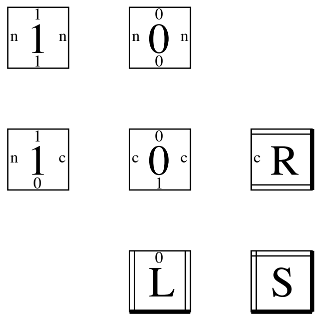
- **given length N, uniquely assemble a N x N square**
- **Temperature τ=1**
  - **Rothemund & Winfree: Lower Bounds:**
    - **has tile complexity at least Ω(N²)**
    - **=> has exact tile complexity: Θ(N²)**

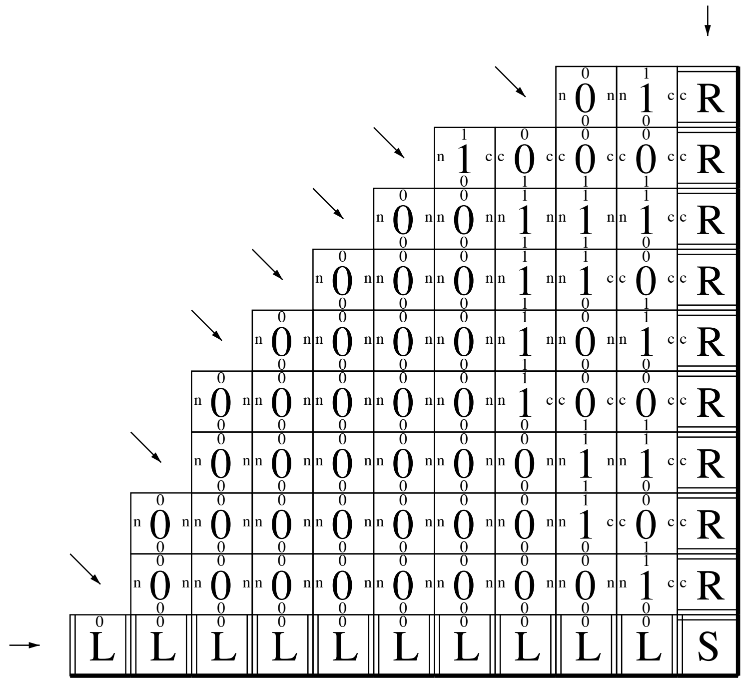
Deterministic Temp τ=2 Square Tiling Complexity



Formation of full squares at $\mathcal{T}=2$. (a) $2N=10$ tiles uniquely produce 5×5 full square. Except for the sides labeled with a circle, each interacting pair of tiles share a unique side label (but we do not label them explicitly as in Figure 2.) (b) $N+4=9$ tiles are used. [Rothemund & Winfree, 2000]

(Temp τ=2) Counter Assembly in 2D



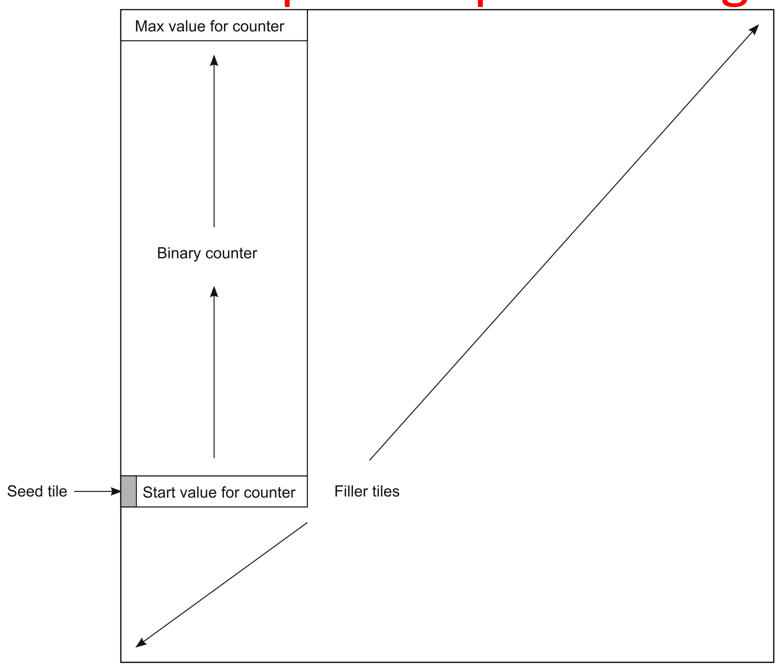


- Assembling a Counter using 7 tiles [Rothemund & Winfree, 2000]
- Can use Counter Assembly to count up to N using O(log N) tiles

Deterministic Temp τ=2 Square Tiling Complexity Results

- **Temperature** τ=2
  - **Rothemund & Winfree: at most O(log(N))**
  - **Adelman: at most O(log(N))/loglog(N)))**
  - **Rothemund & Winfree: at least Ω(log(N)/loglog(N))**
  - **=> has tile complexity: O(log(N))/loglog(N)))**

Deterministic Temp τ=2 Square Tiling Complexity

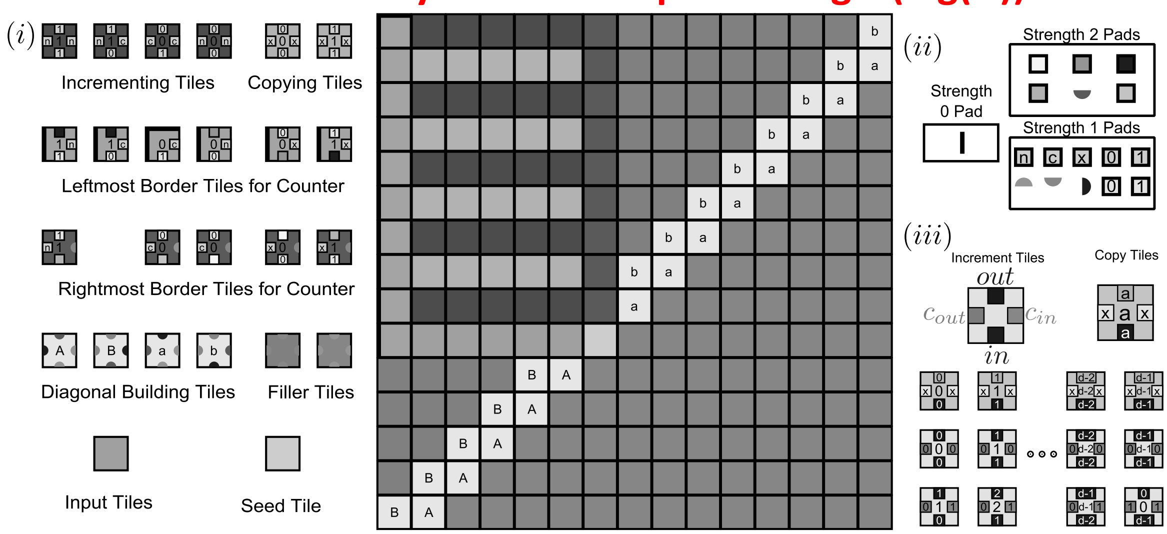


The high level schematic for building an $n \times n$ square using $O(\log n)$ tile types

(Figure from **Patitz, 2012)**

•Rothemund & Winfree: tile complexity at most O(log(N)) for assembly of N x N square

Deterministic Temp τ=2 Square Tiling Complexity
For Self Assembly of a N x N square using O(log(N)) tiles.

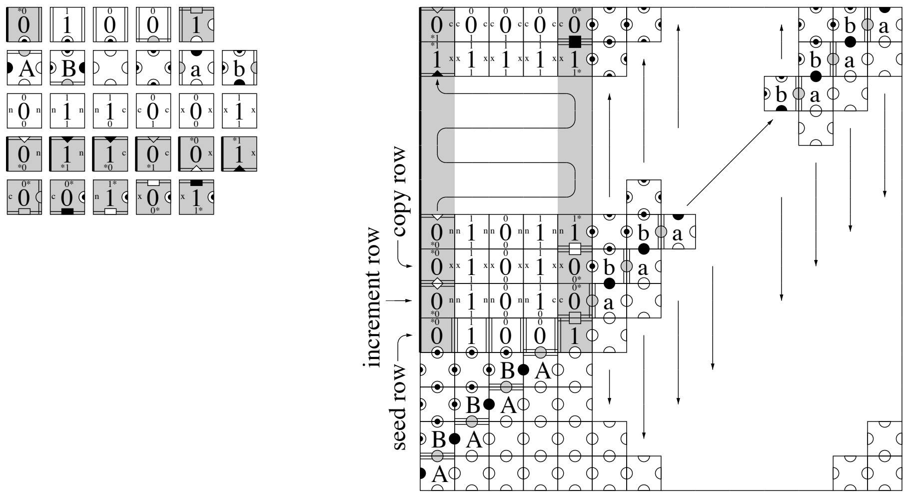


(i) $N \times N$ Square using $\mathcal{O}(\log N)$ tile types. (ii) Pads for $N \times N$ Square using $\mathcal{O}(\log N)$ tile types. (iii) Increment and Copy Tiles for Base d. The border tiles are not shown. The number of tile types is $\Theta(d)$.

(Figure from **Chandran, 2010)**

Rothemund & Winfree: tile complexity at most O(log(N)) for assembly of N x N square

Deterministic Temp τ=2 Square Tiling Complexity
For Self Assembly of a N x N square using O(log(N)) tiles.



Let n= ceiling(log N)

Formation of $N \times N$ square using $O(\log N)$ tiles. Construction starts with an $n-1 \times n-1$ square as in Figure 4b. Here $N = 52, n = 6$ and 28 tiles are used. The construction illustrates the case for even $N - n$; the first row above the seed row is a copy row for odd $N - n$. [Rothemund & Winfree, 2000]

Rothemund & Winfree: Construction of assembly of N x N square with tile complexity at most O(log(N)).

- **The counter assembly (in grey on upper left of N x N square) has height N-n and width n = log(N).**
- **The diagonal continues distance n below the counter assembly, to form square assembly of total width and height (N-n)+n=N.**

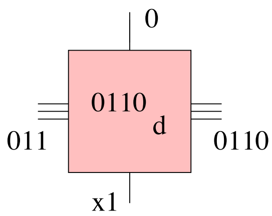
Deterministic Temp τ=2 Square Tiling Complexity
For Self Assembly of a N x N square using O(log(N)/loglog(N)) tiles.

Theorem [Adleman] Assembly of Temp τ=2 Square Tiling set requires at most O(log(N)/loglog(N)) tiles

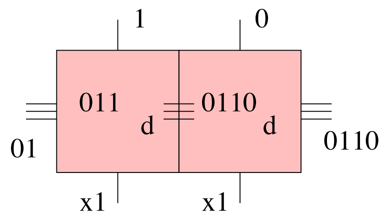
Proof idea:

- **Given N, need to construct tile set that uniquely assembles to an N x N square. Let n=log(N).**
- **Use n/log(n) = log(N)/loglog(N) tiles to encode number N-n by using base b=log(log(N)/loglog(N)) encoding of number N-n.**
- **Form N x N square assembly in 3 stages:**
  - **“Unpack” these log(N)/loglog(N)) tiles : Do base conversion from base log(N) encoding of number N-n to binary encoding.**
  - **Again: use Binary Counter construction to go from binary encoding of N-n to unary encoding of length N-n. The counter assembly (in grey on upper left of N x N square) has height N-n and width n = log(N).**
  - **The diagonal continues distance n below the counter assembly, to form square assembly of total width and height (N-n)+n=N.**

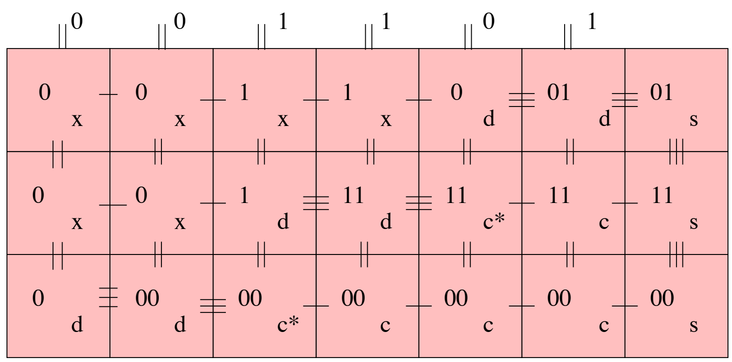
Deterministic Temp τ=2 Square Tiling Complexity
For Self Assembly of a N x N square using O(log(N)/loglog(N)) tiles.



(A) Convert one bit.



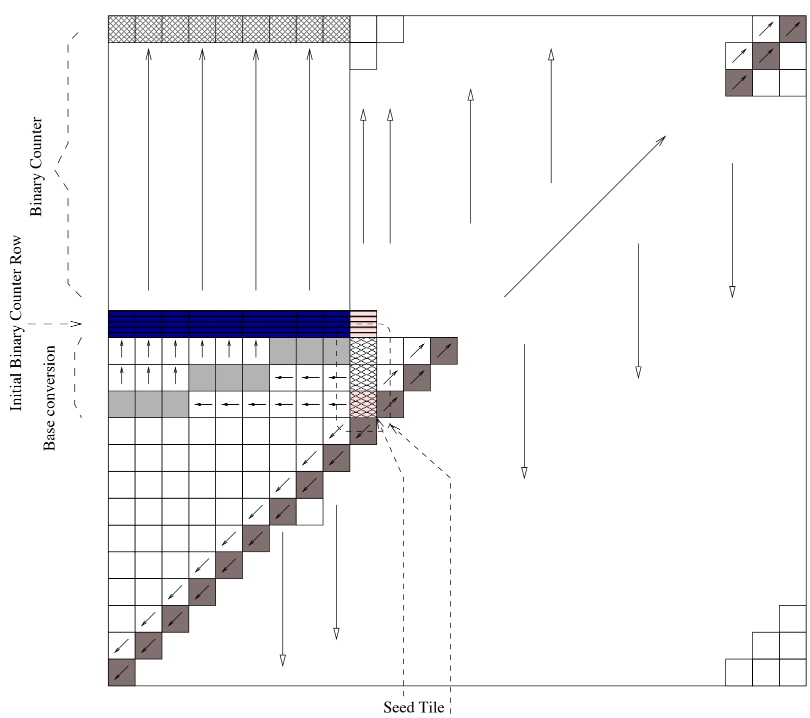
(B) Convert two bits.



“Unpack” encoding of number N to length N assembly

(C) Convert 031 in base 4 to 001101 in base 2.

Deterministic Temp τ=2 Square Tiling Complexity
For Self Assembly of a N x N square using O(log(N)/loglog(N)) tiles.



Assembly communication through diagonal to convert rectangle to square

Lower Bounds on Tile Complexity

- **Consider assembling a line of length n**
  - **Need at least n different tiles (high design complexity)**

    - **Suppose tiles B and F are the same.Then we can “pump” the line segment BCDE into an infinite line**
    - **Are we doomed? No. Can assemble thicker rectangles more efficiently**

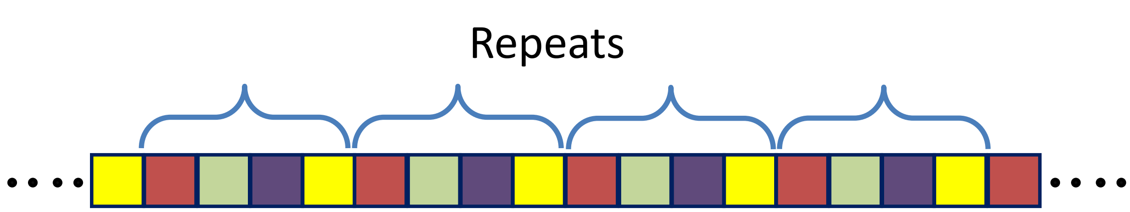
- **Consider assembling an n x n square**
  - **The average Kolmogorov complexity (the smallest program size to produce a desired output) is log n bits**
    - **Thus, k log k = Ω(log n), or k = Ω(log n/log log n)**

Lower Bounds on Tile complexity for Deterministic linear assemblies of length N

- Lower bound in L-TAM is $\Omega(N)$ tile types

- Reason: if a tile repeats, the sequence between the two tiles is pumped infinitely

- Can we modify TAM to get linear assemblies of length N using less than N tile types?



Lower Bounds for Deterministic Tiling Complexity
Temp τ=2 Square For Self Assembly of a N x N square: Matching Lower Bounds

The Kolmogorov complexity K(N) of an integer N with respect to a Turing Machine (TM) is the smallest length TM that encodes N.

- Known result by Kolmogorov: $K(N) = \Theta(\log(N)/\log\log(N))$
  - Proof uses base log(N) encoding of number N

Theorem [Rothemund & Winfree] Temp τ=2 Assembly of Square Tiling requires at least $\Omega(\log(N)/\log\log(N))$ tiles almost always

=> so Temp τ=2 Square Tiling has tight bounds on tile complexity: $\Theta(\log(N))/\log\log(N)))$

Proof by contradiction:

Given a tile set S claimed for assembly of N x N square:

can construct unique assembly of an N x N square

=> so can determine N

Suppose:

Temp τ=2 Square Tiling Complexity is $|S| < c \log(N)/\log\log(N))$ for any constant c.

=> Then can encode N by less than $K(N) = \Theta(\log(N)/\log\log(N))$, a contradiction. QED

Deterministic Temp τ=2 Square Tiling Complexity
For Self Assembly of squate of Approximate Size N x N

Harish Chandrann Nikhil Gopalkrishnan and John Reif, Tile Complexity of Approximate Squares and Lower Bounds for Arbitrary Shapes, Algorithmica, Volume 66, Issue 1 (2013), Page 1-17 (2013)

Theorem [Chandran, Gopalkrishnan, Reif] Approx Temp τ=2 Assembly of Square Tiling size $(1\pm\varepsilon)N$ x $(1\pm\varepsilon)N$ using $O(d+(\log\log(\varepsilon N)/\log\log\log(\varepsilon N))$ tiles where $d=(\log(1/\varepsilon)/\log\log(1/\varepsilon)$

Approximate Assembly Technique: Assemble instead a L x L square where L drops last n-k bits of accuracy:

Will be ε-approximation of an N x N square, where $(1-\varepsilon)N < L < (1+\varepsilon)N$

Given input N, let

$N_1 = \text{floor}((N- (\text{floor}(\log_d N))/2) = b_n b_{n-2} \dots b_0$ base d encoding where $n=(\log_d N_1)+1$ (note is about ½ of N)

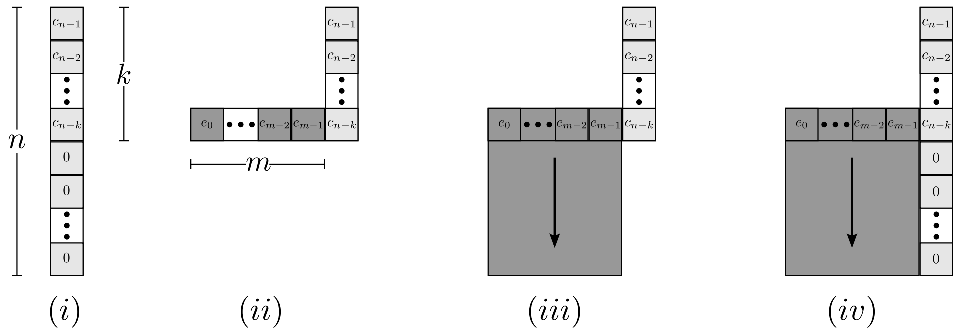
$N_2 = b_n b_{n-2} \dots b_{n-k} 0^{n-k}$ base d encoding with last n-k symbols= 0 and $k = \text{floor}(\log_d(1/\varepsilon))+1$

$N_3 = 1\ 0^{n-k} - N_2 = c_{n-1}\, c_{n-2} \dots c_{n-k}\ 0^{n-k}$ with last n-k symbols = 0

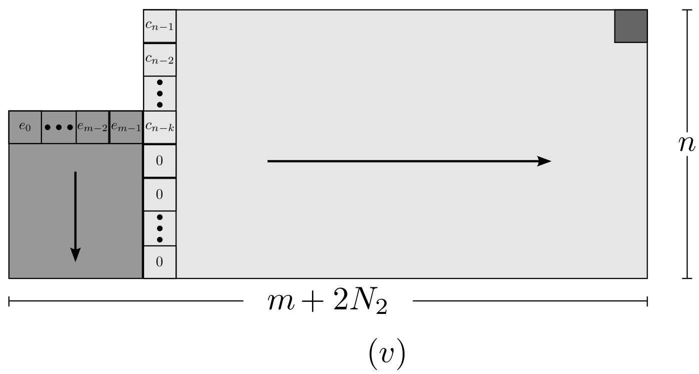
$m= \text{ceiling}(\log(n-k)/\log\log(n-k))$

Then Assemble a L x L square where L is just over size $(2N_2+n)$

Approximate Deterministic Temp τ=2 Square Tiling



[Chandran, Gopalkrishnan, Reif] Assembly of Minor & Major Counters



Components of the construction: (i) *Seed column* for the major counter. (ii) L-shaped seed assembly. (iii) Assembly of the minor counter. (iv) Completing the *seed column* using the 0 tile type. (v) Assembly of the major counter.

Approximate Deterministic Temp τ=2 Square Tiling

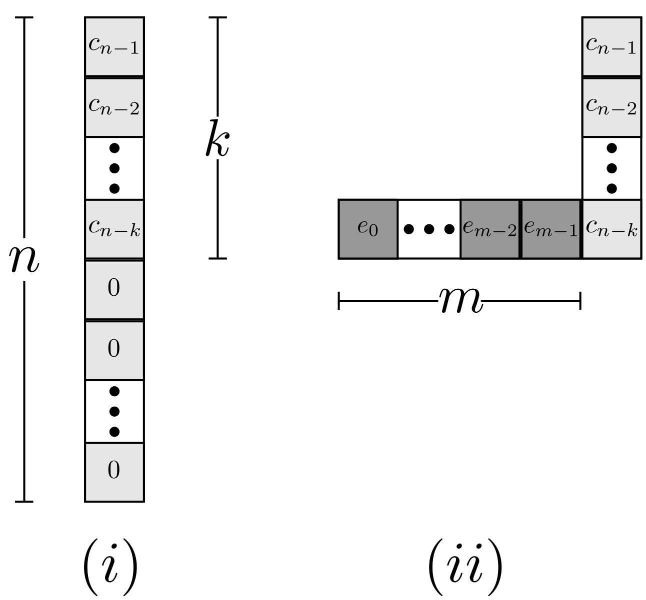


Figure [Chandran, Gopalkrishnan, Reif] L-shaped seed assembly

Horizontal row is seed for base m= ceiling(log(n-k)/loglog(n-k)) counter with height n-k.

Vertical column has k vertical tiles to encode $c_{n-1}\, c_{n-2} \ldots c_{n-k}$

Approximate Deterministic Temp τ=2 Square Tiling

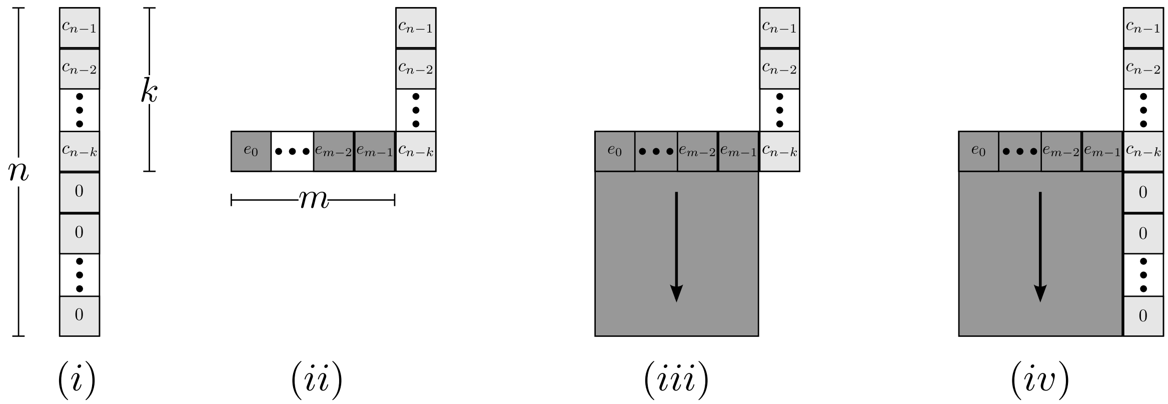


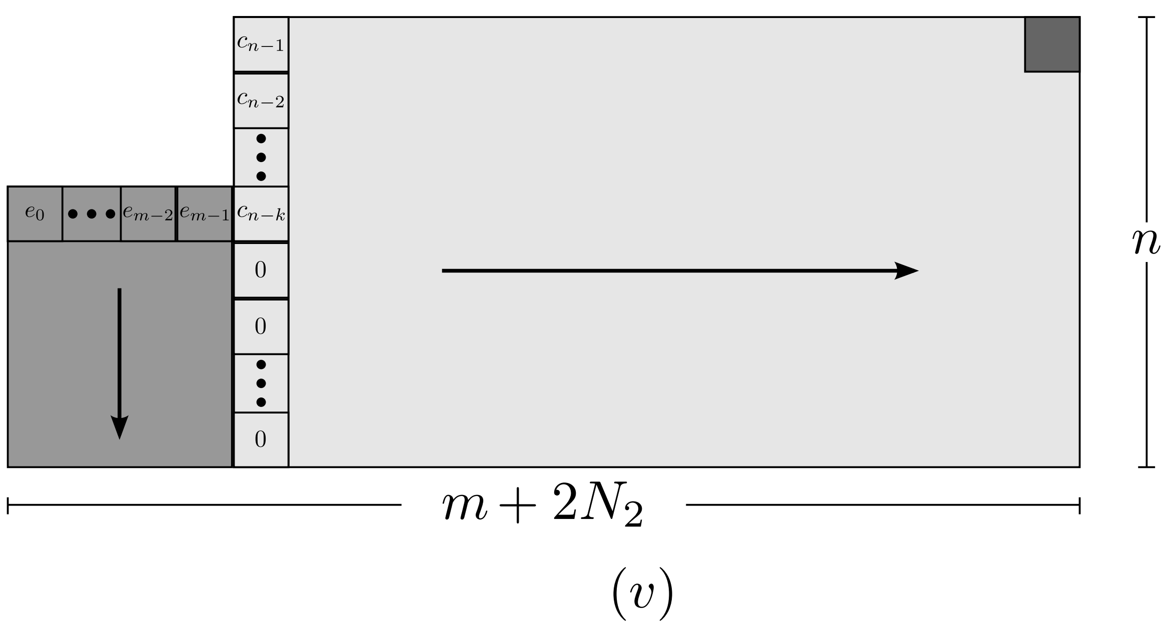
Figure [Chandran, Gopalkrishnan, Reif] Assembly of Minor Counter from L-shaped seed assembly: using m tile types

Rectangle width m and height n-k (excluding seed row):

Horizontal row is seed for base m= ceiling(log(n-k)/loglog(n-k)) counter with height n-k.

Vertical column has k vertical tiles to encode $c_{n-1}\, c_{n-2} \ldots c_{n-k}$

Approximate Deterministic Temp τ=2 Square Tiling



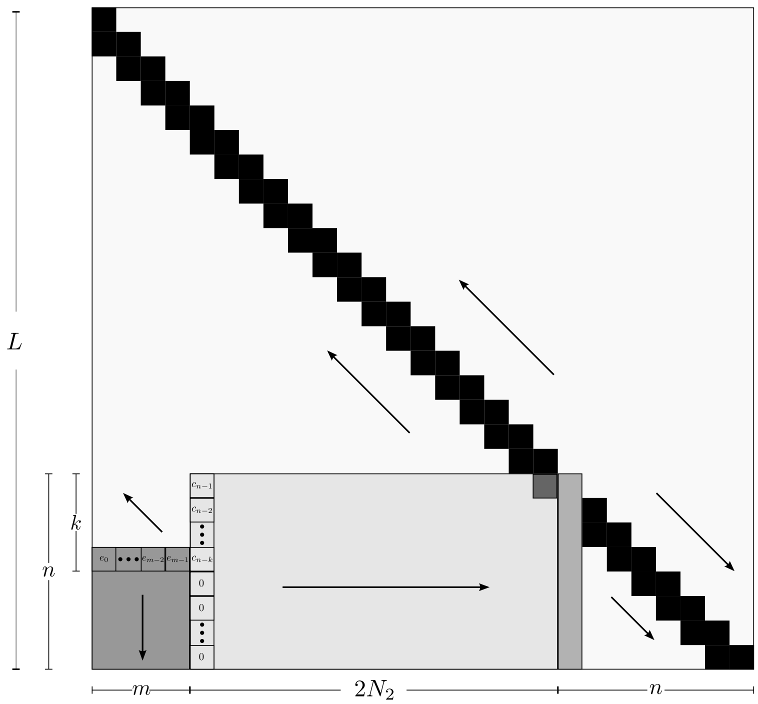
[Chandran, Gopalkrishnan, Reif] Assembly of Major Counter:

Is n x $2N_2$ rectangle

Uses column representing d-ary encoding of

$N_3 = 1\,0^n - N_2 = c_{n-1}\, c_{n-2} \ldots c_{n-k}\, 0^n$ to count up to $1\,0^n$ (with n 0s) in base d

Approximate Deterministic Temp τ=2 Square Tiling



[Chandran, Gopalkrishnan, Reif]
Diagonal and filler tiles complete approximate square of length $L = 2N_2 + m + n$

Approximate Deterministic Temp τ=2 Square Tiling

Theorem [Chandran, Gopalkrishnan, Reif] Approx Temp τ=2 Assembly of Square Tiling size $(1\underline{+}\varepsilon)N$ x $(1\underline{+}\varepsilon)N$ requires $\Omega(d+(\log\log(\varepsilon N)/\log\log\log(\varepsilon N))$ tiles almost always, where $d=(\log(1/\varepsilon)/\log\log(1/\varepsilon)$

Case 1: ε > 1/4: Lower bound is within constant factor of exact case

Case 1: ε $\le$ 1/4: use **Kolmogorov complexity lower bound argument**

Proof by contradiction:

Given a tile set S claimed for ε-approximate assembly of N x N square:

Can construct unique assembly of an L x L square

Which is ε-approximation of an N x N square, where $(1-\varepsilon)N < L < (1+\varepsilon)N$

So can determine first n=floor(L)+1 > floor(log(N)) bits of N

=>Can show violates Kolmogorov complexity lower bound for encoding n bit number.

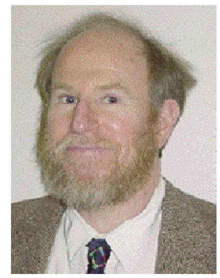
QED

Randomized Tile Complexity of Linear Assemblies

Harish Chandran, Nikhil Gopalkrishnan, John Reif

- We extend TAM to incorporate stochastic behavior
- We study linear assemblies in this new model:
  The Probabilistic Tile Assembly Model (PTAM)





Harish Chandran, Nikhil Gopalkrishnan, and John Reif, The Tile Complexity of Linear Assemblies, SIAM Journal of Computation (SICOMP), Society for Industrial Mathematics, Vol. 41, No, 4, pp. 1051-1073, (2012).

Multiple Possible Outputs of tiling systems For Randomized Assemblies

- We relax this constraint
- Result: many final shapes can be produced
- Many outputs for an instance of a problem

Probabilistic Tile Assembly Model (PTAM)

- **Make tile attachments non-deterministic**
- **Multiple tile types can attach to a given position in a partial assembly**

Probabilistic Tile Assembly Model (PTAM)

- **We allow the tile set to be a multiset, i.e., each tile type can occur multiple times**
  - **Example: {A,B,C,C,C,C,D}**
- **The multiplicity of each tile type indicates the tile type's concentration**
  - **Example: {1:1:4:1}**

Probabilistic Tile Assembly Model (PTAM)

- **At each stage of the assembly and at each growth position, a tile is chosen from the multiset with replacement**

- **If the tile can bind at that site, it does, else another tile is chosen until no tile can be added**

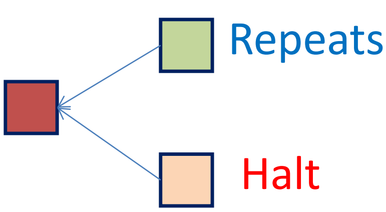
- **Output of a tiling system is a set of shapes**

- **For linear assemblies, we define the output of a tiling system as the *expected length* of linear assemblies it produces**

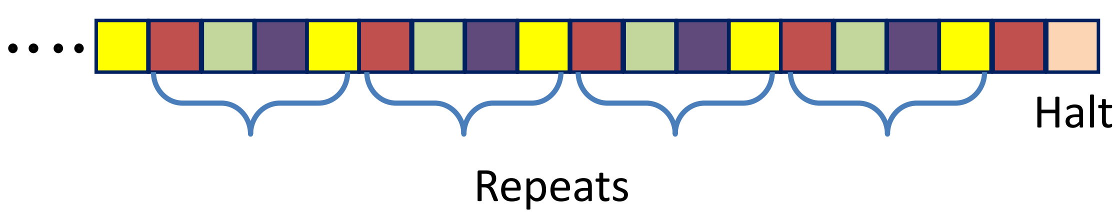
How does this affect the lower bound of linear assemblies?

- More than one tile can attach at a given spot
  - So repeats can occur, yet the system can halt

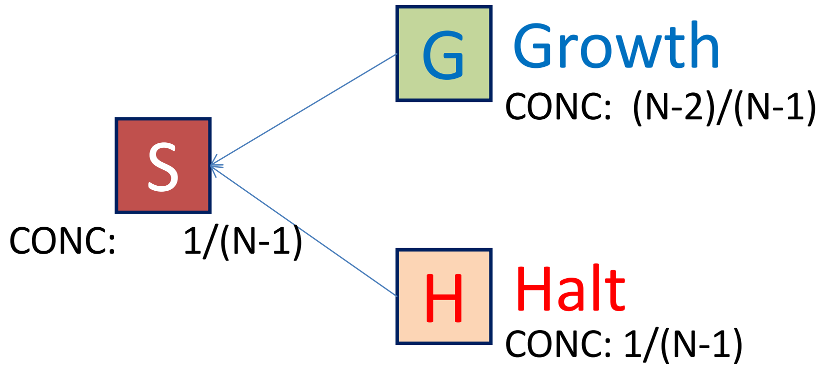
- Notation: Arrows indicate probabilistic tile attachment with equal probability



Both the tiles can attach to the red tile, probability of attachment depends on relative concentration



Example: a three tile PTAM system for linear assemblies of expected length N



Tile Multiset for the above system: $\{S, \underbrace{G, G, \ldots, G}_{n-2}, H\}$

More on tile multisets

- By making the tileset a multiset, we implicitly encode information about the concentration of tile types
- Cardinality of a tile multiset is a true indicator of the information the tile set encodes
- Cardinality of a tile multiset is the descriptional complexity of the shape
- Though the previous example had only 3 tile types, the tile multiset had N tiles in it
- *No improvement from deterministic scenario*

Linear assemblies of expected length N in PTAM

- We first show a construction using $O(\log^2 N)$ tile types

- Then we show a more complex construction using $O(\log N)$ tile types

- Next we show a matching lower bound $\Omega(\log N)$ tile types are required to build linear assemblies of expected N

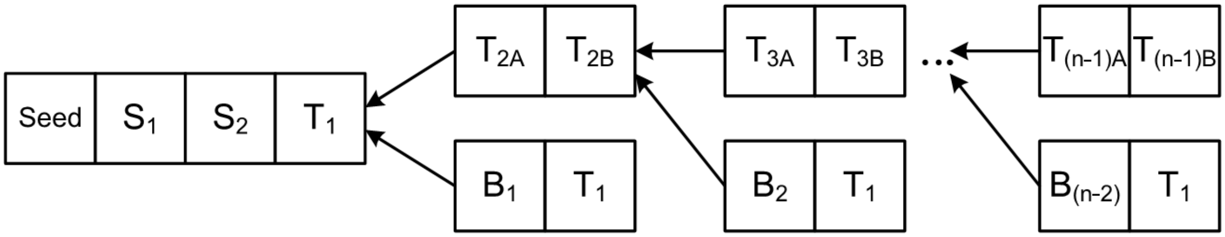
- Methods for constructing linear assemblies of length N with high probability using $O(\log^3 N)$ tile types for infinitely many N

Linear assemblies of expected length N using $O(\log^2 N)$ tile types

- We show how to construct linear assemblies of expected length N using $O(\log N)$ tile types for any N that is an exact power of 2

- We then describe a method to extend this construction to all N using $O(\log^2 N)$ tile types

Powers of two construction



- Restarts with addition of B_iT_i tile complex after T_{iB}
- Goes forward with addition of $T_{(i+1)A}T_{(i+1)B}$ tile complex after T_{iB}
- Each happens with equal probability
- Process akin to tossing a fair coin till we see n-2 consecutive heads

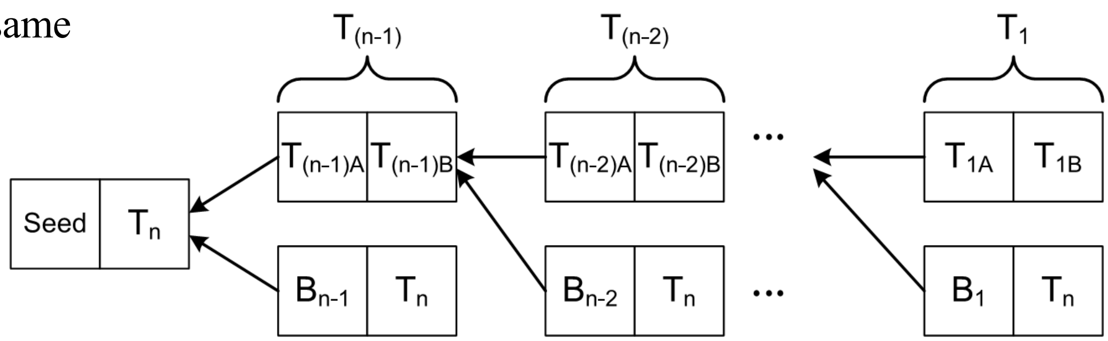
•Probabilistic branching system shown above using tile multiset of cardinality O(n) has: Expected length = 2^n

Linear assemblies of expected length N using O(log^2 N) tile types

- We extend this to any N by:
- Considering the binary representation of $N = b_0 2^0 + b_1 2^1 + b_2 2^2 + \ldots b_n 2^n$, where n = floor(log(N)).
- Constructing assemblies of expected length equal to numbers represented by each 1 in the binary representation of N
  - Each of these is a '*powers of two*' construction
- Deterministically concatenating these assemblies
- Each subassembly requires O(log N) tile types and there are a maximum of O(log N) of these
- Thus total number of tile types = $O(\log^2 N)$

Linear assemblies of expected length N using O(log N) tile types

- ***Key idea***: E[# T_{k-1} appears] = ½ E[#T_k appears]
- Restart bridge B_{k-1} appears other half of the time
- We use this property and make some links deterministic
- ***Observation***: Every time we branch, expected number of times the next tile appears is halved, if we don't branch, the expectation remains the same

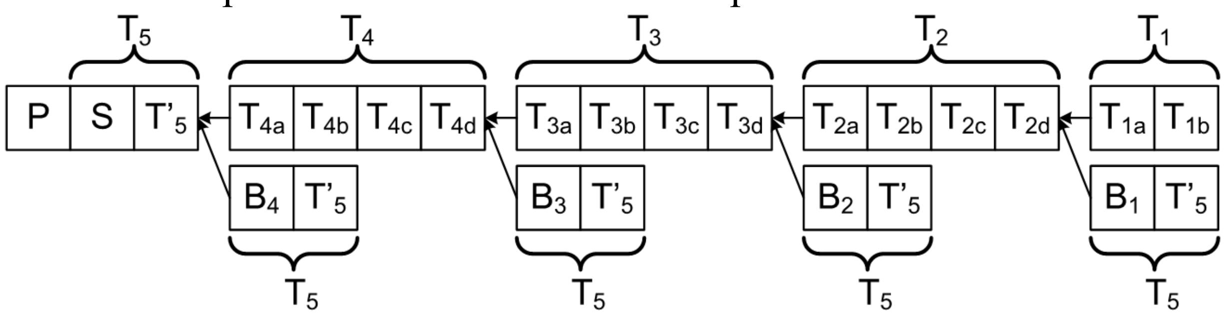


Linear assemblies of expected length N using O(log N) tile types

- ***Key idea***: Any number N can be written in an alternate binary encoding using {1,2} instead of {0,1}
- For example 45 = $(101101)_{\{0,1\}}$ = $(12221)_{\{1,2\}}$
- $1x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 1x2^0 = 45$
- $1x2^4 + 2x2^3 + 2x2^2 + 2x2^1 + 1x2^0 = 45$
- ***Observation***: The number of bits in this new encoding of N is at most log N.
- We illustrate this technique using an example

Example: Linear assemblies of expected length 91

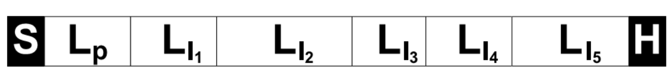
- To get 91, we find the alternate encoding of floor(91/2) =45
  - $45 = (12221)_{\{1,2\}}$
- For the bits that are 2, we construct complexes of size 4
  - Deterministic links, expectation stays same
- For bits that are 1 we construct complexes of size 2
  - Probabilistic links, expectation is halved
- We add a prefix tile if N was odd to compensate for the floor



Number of tile types required : O(log N)

Lower bounds for linear assemblies

- **Can we do better than O(log N)?**
  - **NO!**
- Proof sketch:



- Split each run of a tile set with n tile types into
  - Intermediates
  - Prefix
- Simulate each segment using fewer number of tiles
- Can show through a recursive argument on each of these segments that maximum length is $O(2^n)$

Lower bounds for linear assemblies

• Thus, for each N, the cardinality of tile multiset to construct a linear assembly of expected length N is $\Omega(\log N)$

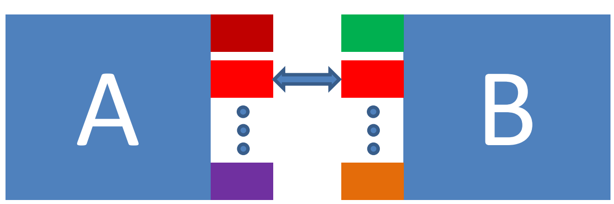
• Notice that this bound is true for all N

• Stronger than the usual Kolmogorov complexity based lower bounds that holds only for almost all N

k-pad Tiles

• A simple extension to PTAM is the k-pad PTAM system

• Each tile now has k-pads on each side



• Possible implementation via DDX or origami

• This allows more choices for binding with a tile

• Tiles bind if at least one of their corresponding pads match

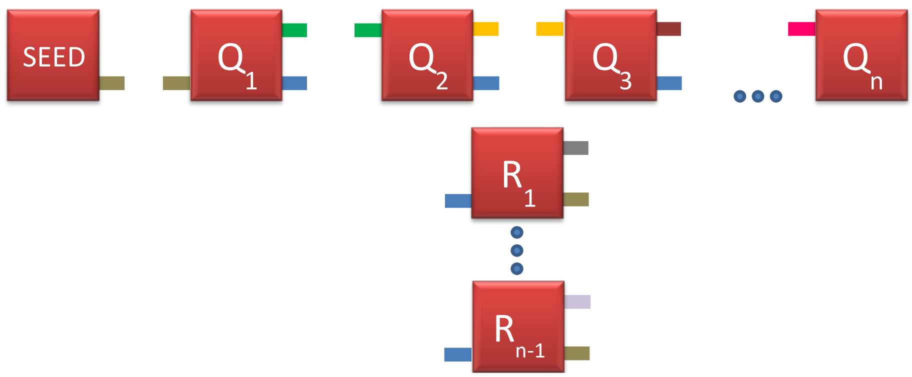
•Note that the descriptional complexity in 2-pad PTAM is still the cardinality of the tile multiset

Linear assemblies of expected length N using $O_{i.o}$(log N/ log log N) k-pad tile types

- The system shown below is akin to tossing a biased coin (Head : Tail :: 1 : n) till we get n successive heads

- Expected number of tosses for this : n^{2n}

- We can get linear assemblies of expected length N using a tile multiset of cardinality O(log N/ log log N) 2-pad tiles for infinitely many N



Lower bounds for k-pad systems

- Can we do better than $O_{i.o}$(log N/ log log N)?
  - NO!

- Proof sketch:

- Convert any k-pad tile system into a graph
  - Tiles -> vertices
  - Possible attachments -> edges

- Self-assembly is a random walk on the graph

- Expected length of the assembly is the expected *time T to first arrival* to the vertex for the halting tile

- This can be solved as a system of linear equations

- Bound first arrival time T by a ratio of determinants of size $N^{O(\log N)}$

Lower bounds for k-pad systems

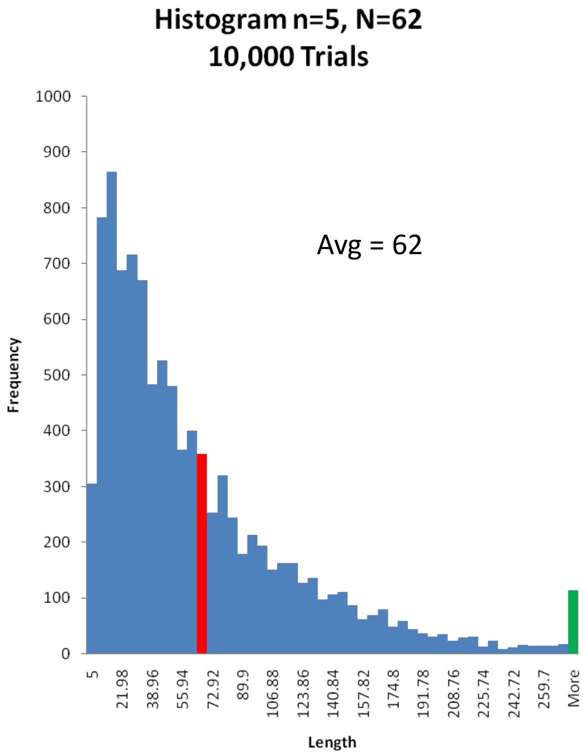
- Thus, for each N, the cardinality of tile multiset to construct a linear assembly of expected length N using k-pad tiles for any given k is $\Omega(\log N / \log \log N)$

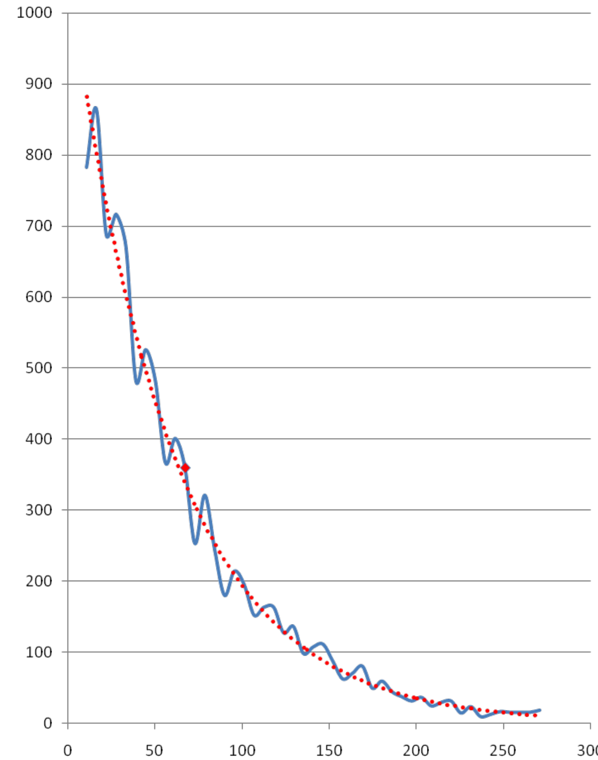
- As before, this bound is true for all N
  - Stronger than the usual Kolmogorov complexity based lower bounds that holds only for almost all N

Distribution and tail bounds

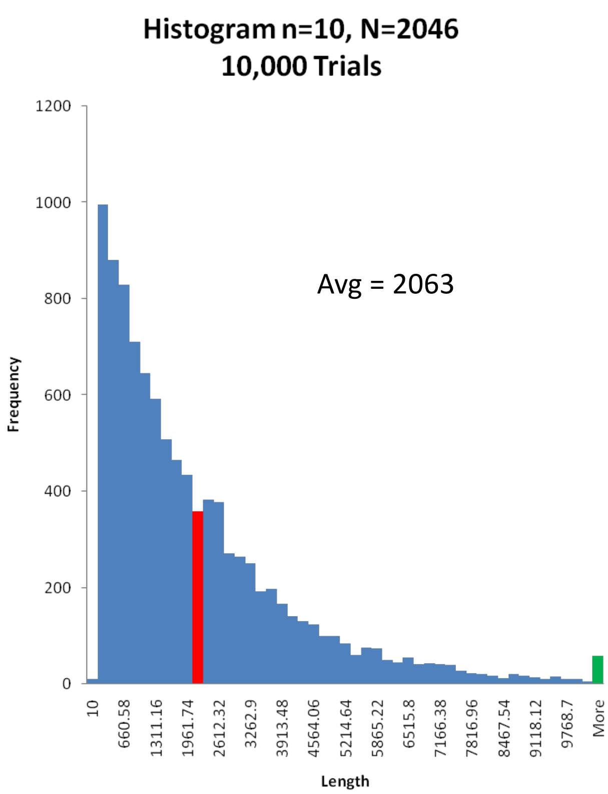
- We constructed linear assemblies of given length in expectation
  - What about the distribution of lengths?
- We can concatenate k assemblies each of expected length N/k deterministically to improve tail bounds
- By central limit theorem, as k grows large, the distribution approaches the standard normal distribution
- We get an exponentially dropping tail for a multiplicative increase in the tile set cardinality
- If k = N, we get a deterministic assembly (degenerate distribution)
- This is illustrated in the following examples

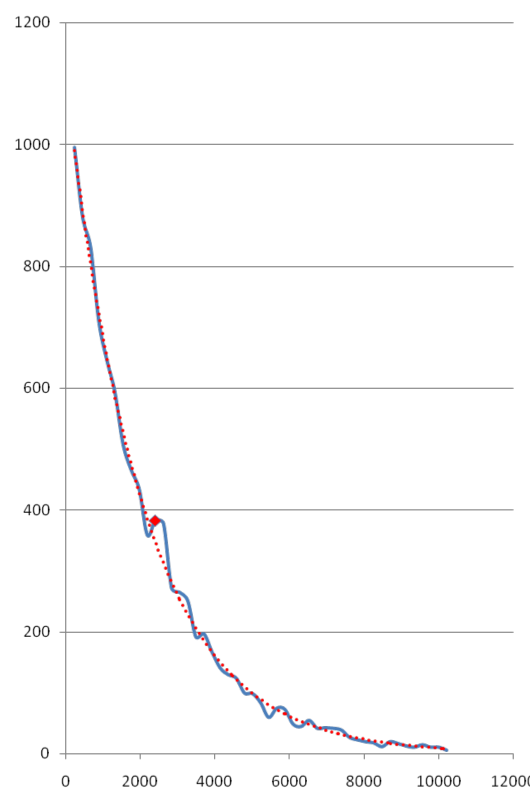
Example: 5 consecutive heads



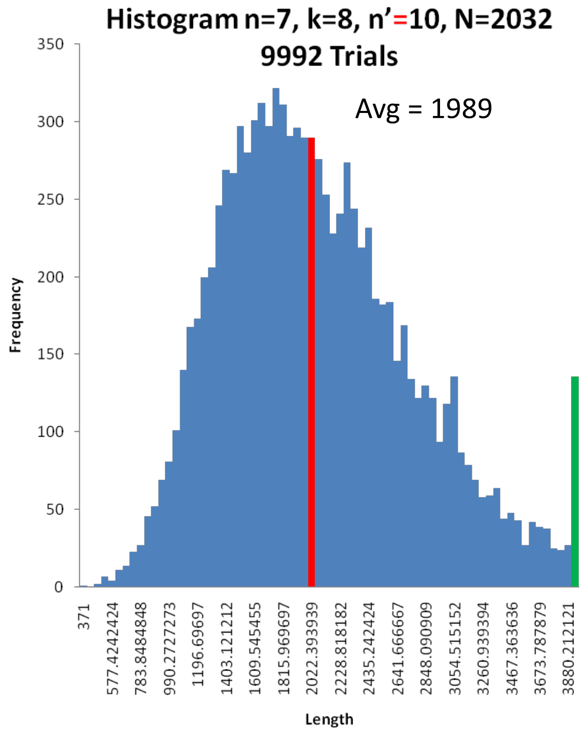


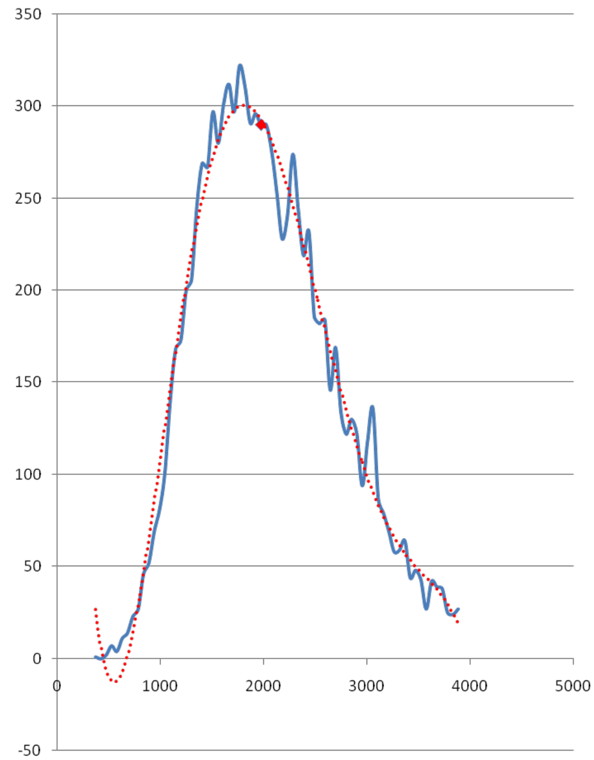
Example: 10 consecutive heads



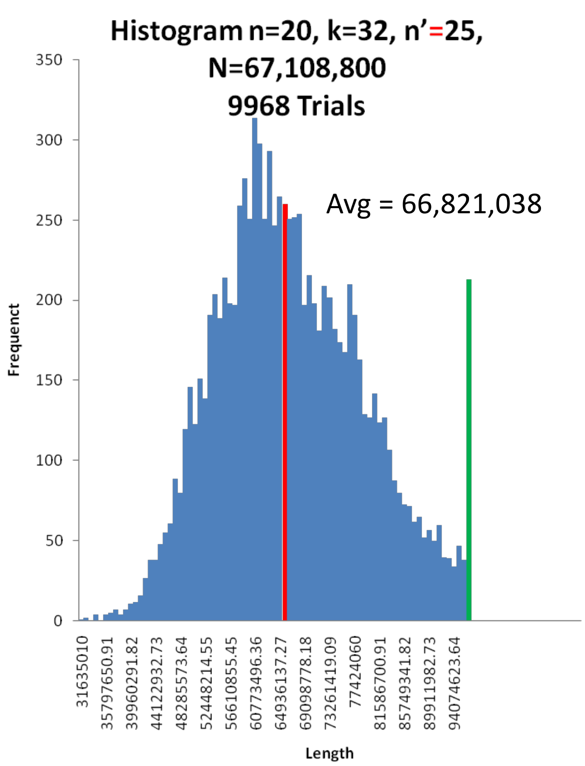


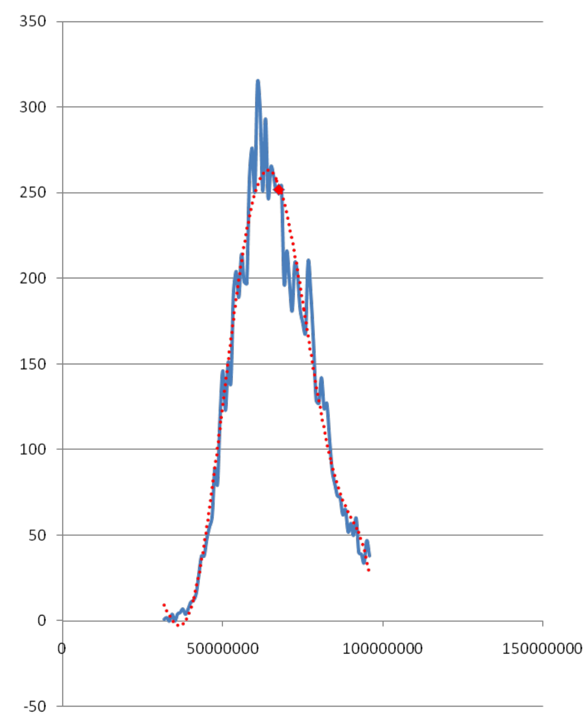
Example: 8 concatenations of 7 consecutive heads (similar to 10 consecutive heads)





Example: 32 concatenations of 20 consecutive heads (similar to 25 consecutive heads)





Summary

- **Introduced the Probabilistic Tile Assembly Model**
  - **k-pad systems**

- **Studied the tile complexity of linear assemblies**

- **Showed how to construct linear assemblies of expected length N using O(log N) tile type**

- **Proved that this is the best one can do by deriving a matching lower bound**

- **Proved analogous results for k-pad systems**

- **Provided a method to improve tail bounds**

Future directions

- **Tightened tail bounds**

- **Running time analysis of all the systems described earlier**

- **Error correction in PTAM systems for linear assemblies**

- **Experimental Implementation of the DNA tile assemblies in the laboratory**