Alternative Tile Assembly Models and Complexity Results

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- All screenshot pictures are from original papers.

Outline

- Multiple Temperature model and complexity results.
- Staged assembly model and complexity results.
- Flexible glue model, time-dependent glue model and complexity results.

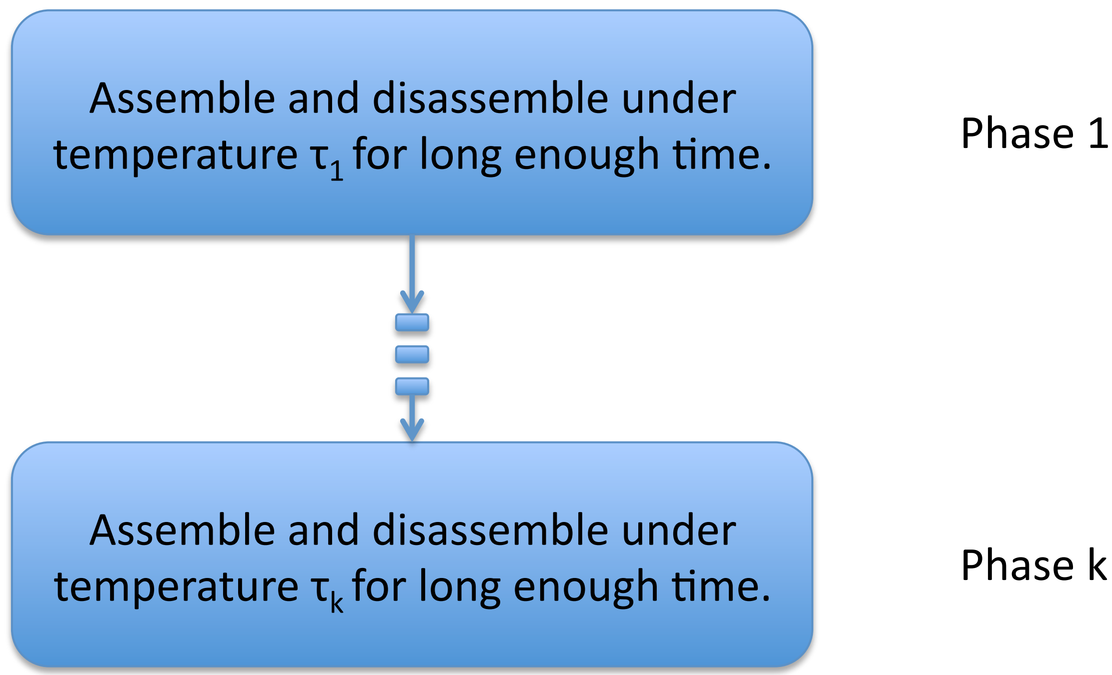
Multiple Temperature Model

- A tile system $S = <T, g, s, \{\tau_i\}_{i=1}^{k}>$ under multiple temperature model[Aggarwal et al. SODA 2004].
- $\{\tau_i\}_{i=1}^{k}$ is temperature sequence.
- Temperature complexity: the number of items in the temperature sequence.

Assembly Process

- One pot reaction
- Multiple phases: each phase has its own temperature.
- Assemble and disassemble

Assembly process



The terminal product of phase k is the terminal product of this k-temperature system.

Basic Tool

- Bit-Flipping[Kao et al. SODA 2006].

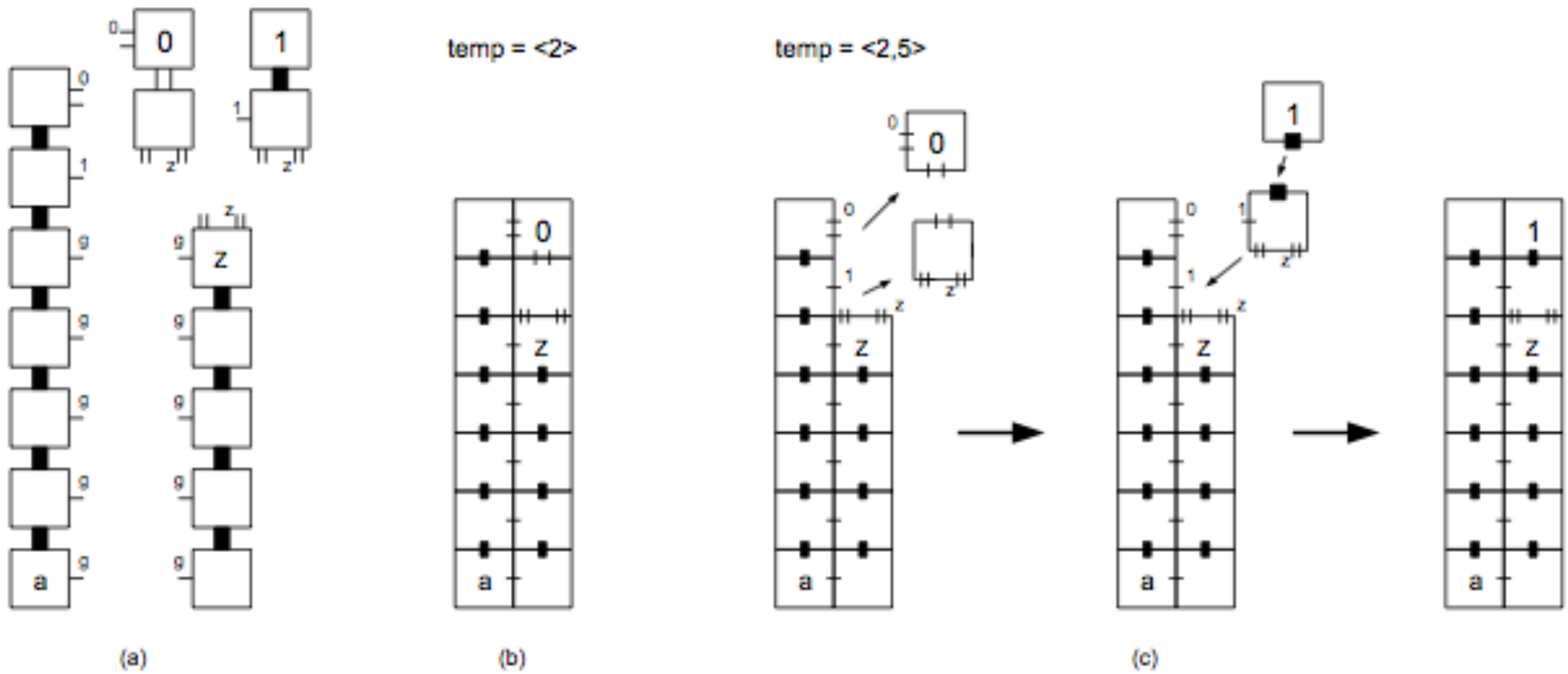
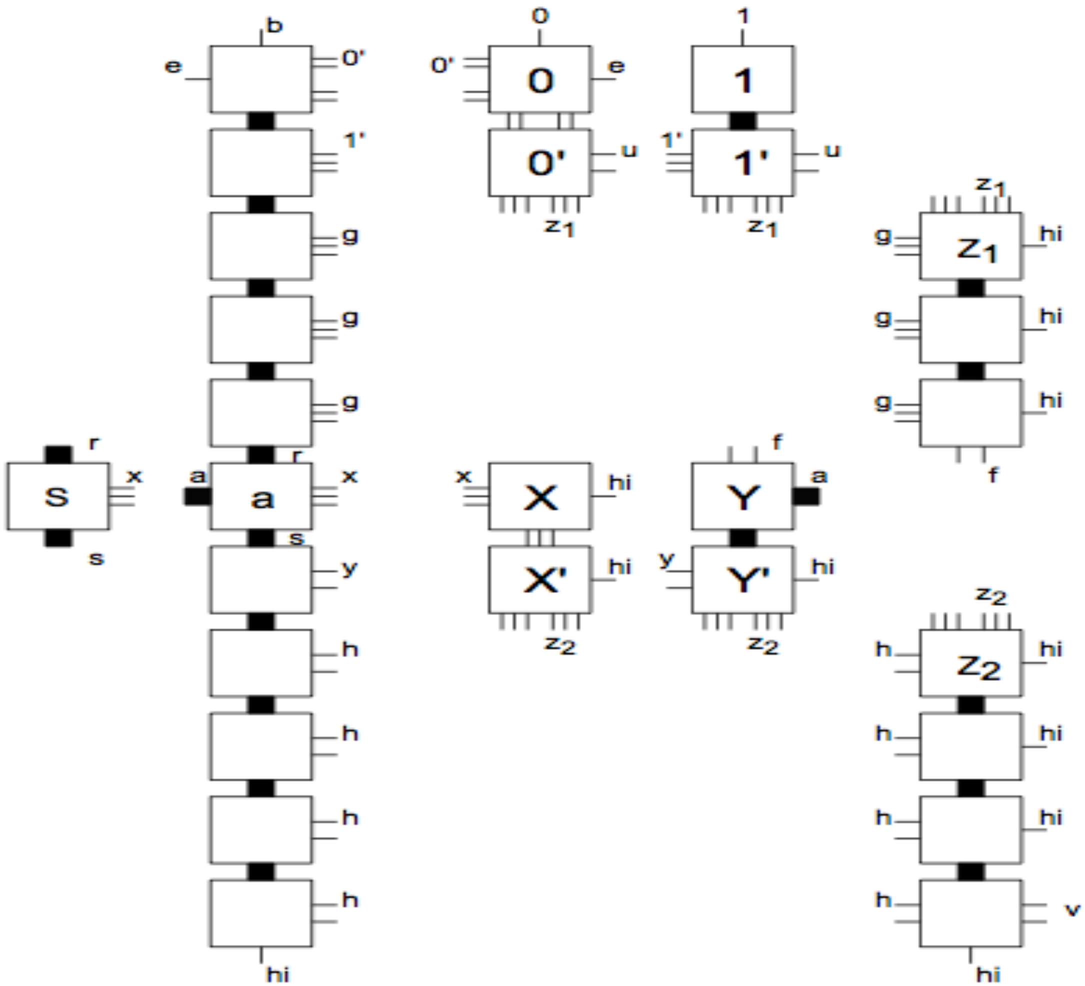


Figure 1: (a) Tiles that implement the bit flip gadget. The number of lines on the side of a tile represents the strength of the corresponding glue. The dark black line denotes a strength 5 glue. (b) At temperature 2, with tile a as the seed tile, the bit flip gadget uniquely assembles a supertile with the 0 tile in the top right corner. (c) By raising the temperature to 5, the 0 tile breaks off and is replaced by the 1 tile.

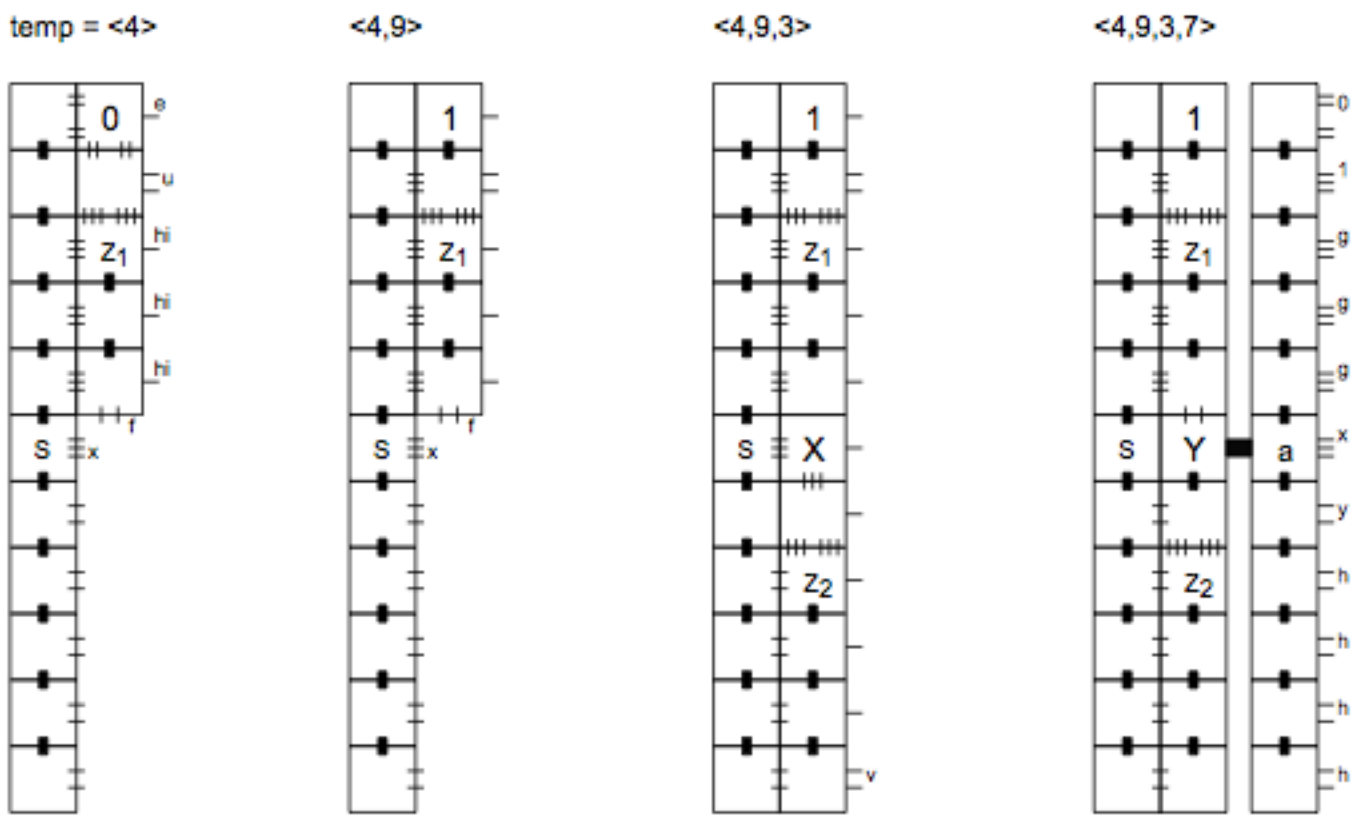
Binary Strings with O(1) Tiles

- Tile set[Kao et al. SODA 2006]:

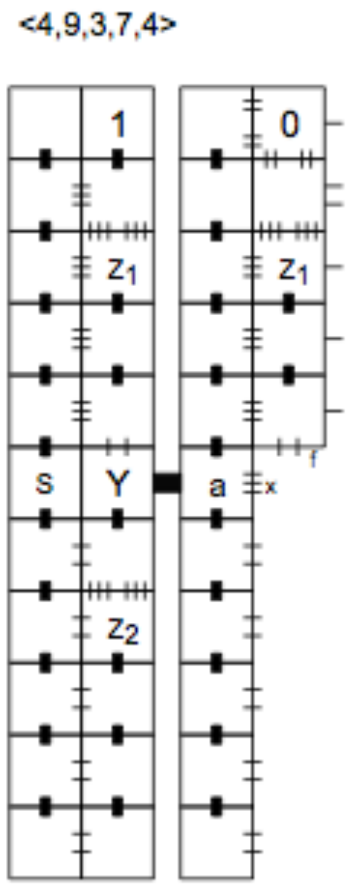


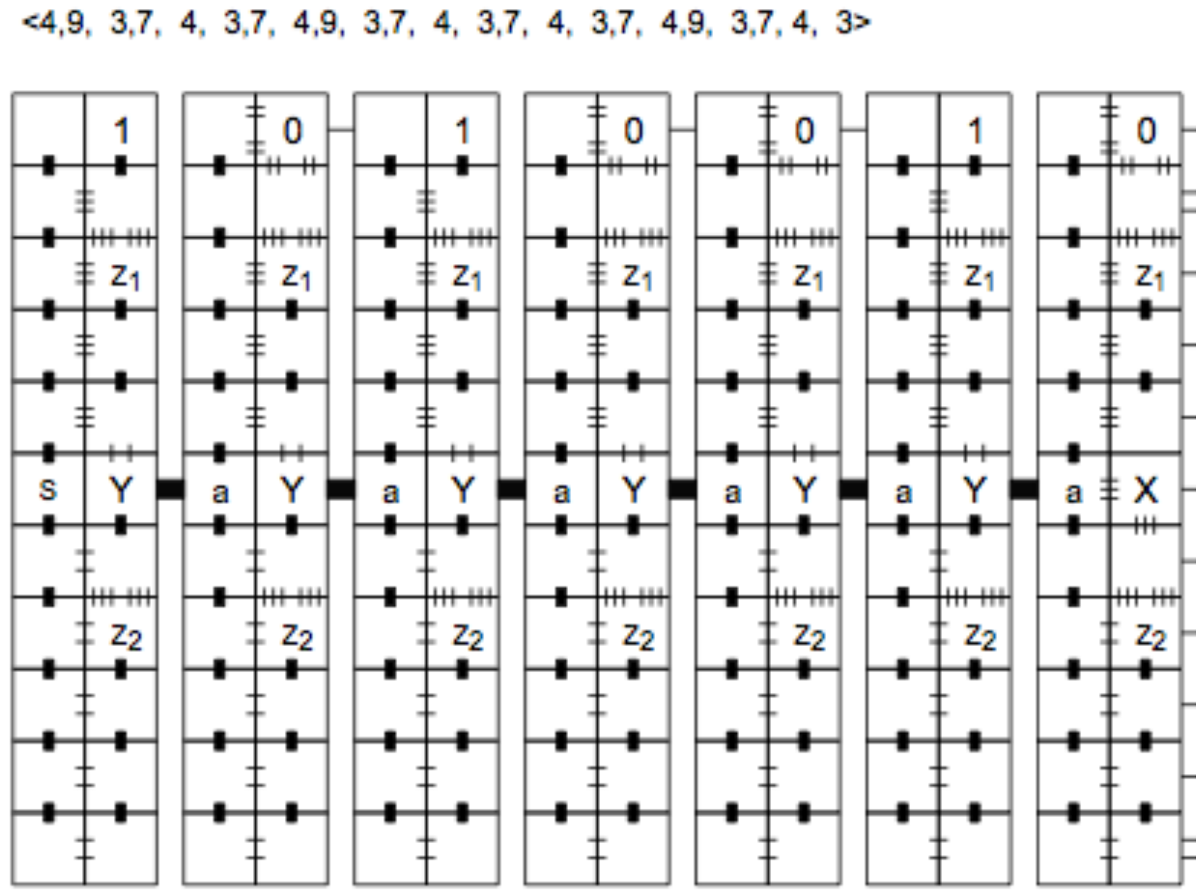
Binary Strings with O(1) Tiles

- Example: 1010010

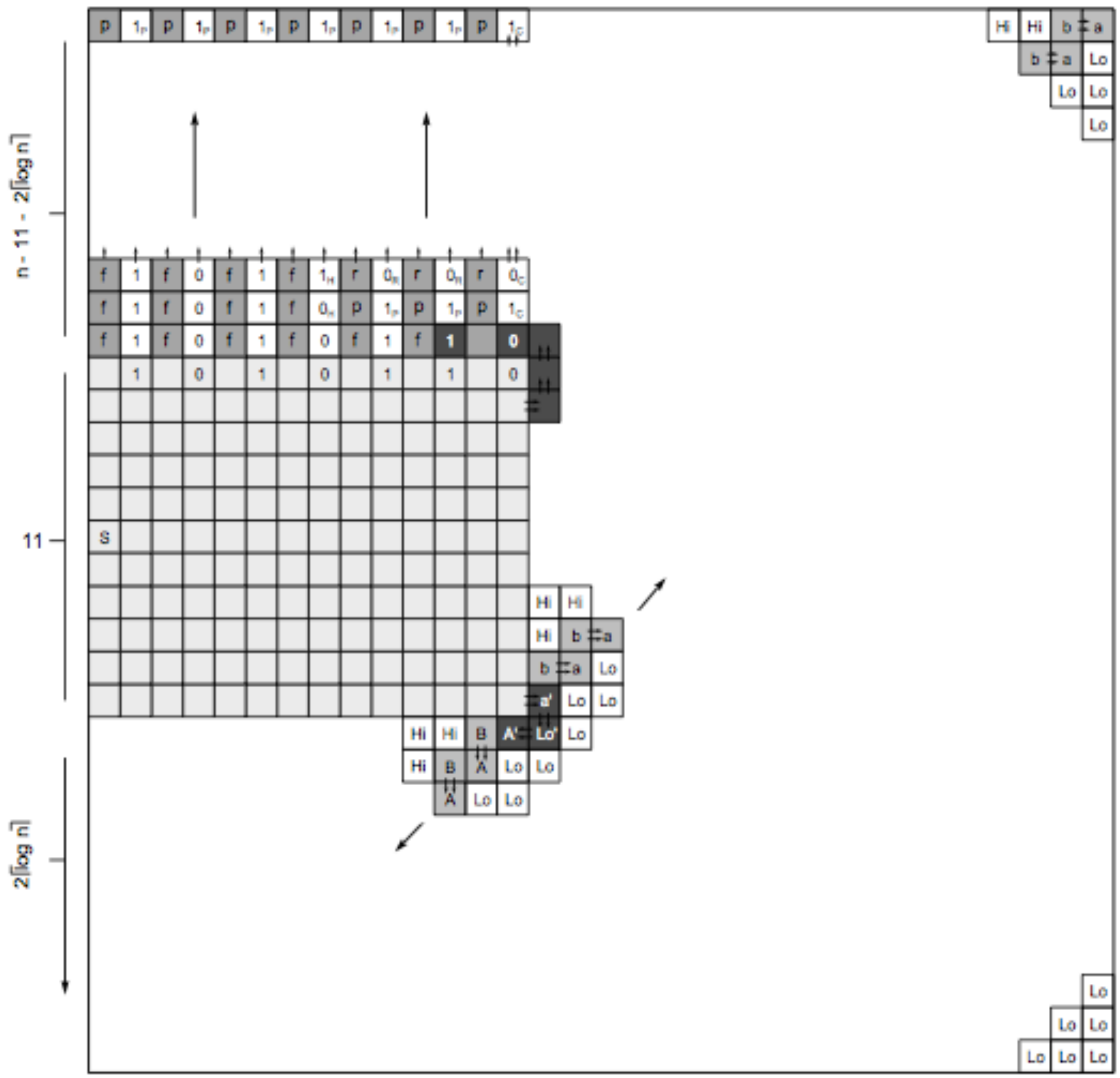


Binary Strings with O(1) Tiles





Building n×n Square in O(1) Tiles



Staged Assembly Model

- A tile system $S=<M_{r,b}, \{T_{i,j}\}, \{\tau_{i,j}\} >$ under staged assembly model[Demaine et al. Nat Comput 2008].
- Assumption: *merge, split, extract*
- $M_{r,b}$: r-stage b-bin mix graph M.
- $\{T_{i,j}\}$: $T_{i,j}$ is the tile set of $bin_{i,j}$.
- $\{\tau_{i,j}\}$: $\tau_{i,j}$ is the temperature of $bin_{i,j}$.
- Stage complexity, bin complexity

Example of Staged Assembly System

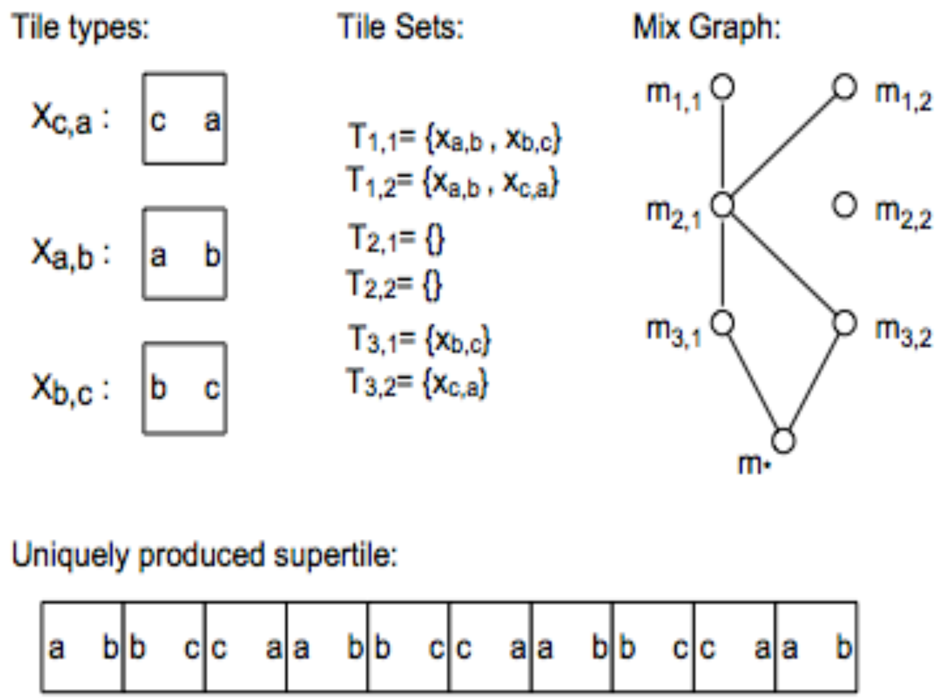


Figure 1: A sample staged assembly system that uniquely assembles a $\mathbf{1} \times \mathbf{10}$ line. The temperature is $\tau = 1$, and each glue $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ has strength $\mathbf{1}$. The tile, stage, and bin complexities are $\mathbf{3}$, $\mathbf{3}$, and $\mathbf{2}$, respectively.

Assembly of 1×n lines

- Special case: there is a planar temperature-1 staged assembly system that uniquely produce a(full connected) 1×2^k line using 3 glues, 6 tiles, 6 bins, and O(k) stages [Demaine et al. Nat Comput 2008].
- Planar system:

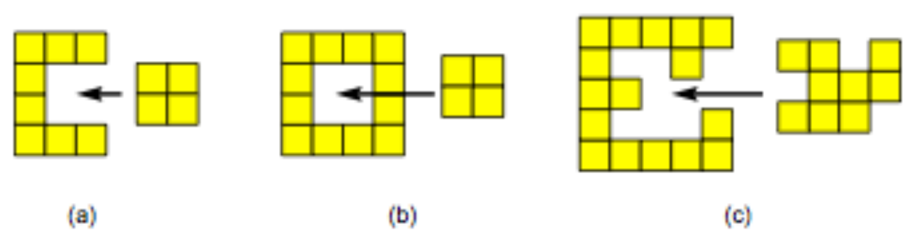


Figure 2: All three assemblies are permitted under the basic model. However, only assembly (a) is permitted under the planarity constraint.

Assembly of 1×n lines

- Strategy: divide and conquer

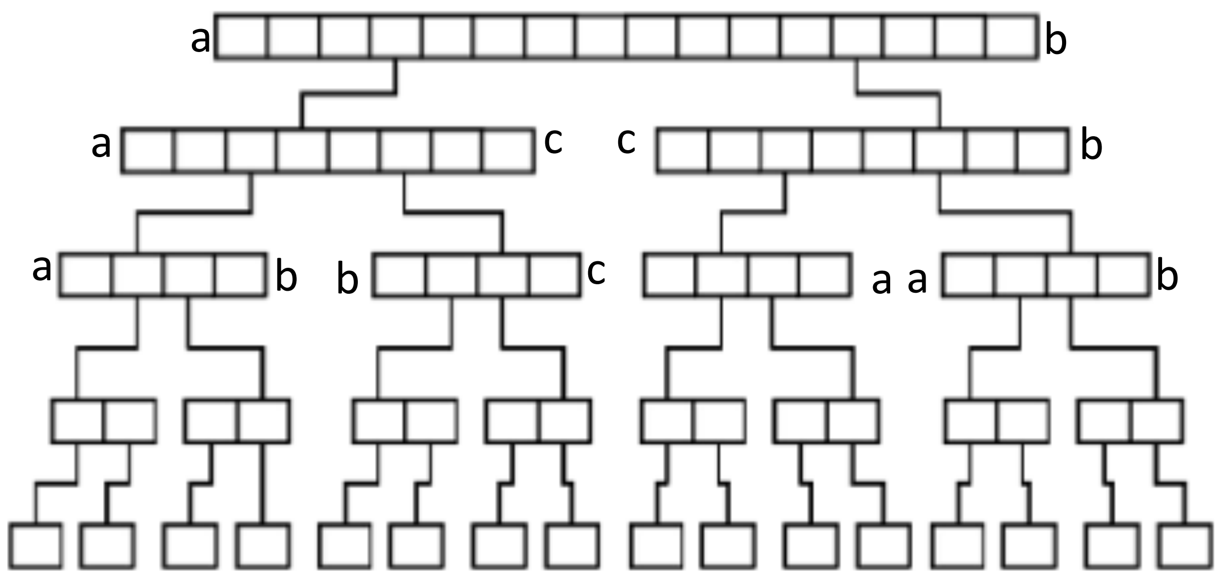


Figure 3: Decomposition tree for 1×16 line.

Assembly of 1×n lines

- General case: standard trick, build 1×2^i line for the i*th* bit of binary encoding of n if the i*th* bit is 1.
- Theorem: there is a planar temperature-1 staged assembly system that uniquely produce a(full connected) 1×n line using 3 glues, 6 tiles, 7 bins, and O(logn) stages [Demaine et al. Nat Comput 2008].

Assembly of n×n Squares

- Theorem: there is a planar temperature-1 staged assembly of a full connected n×n square using 9 glues, O(1) tiles, O(1) bins, and O(logn) stages[Demaine et al. Nat Comput 2008].

Jigsaw Technique

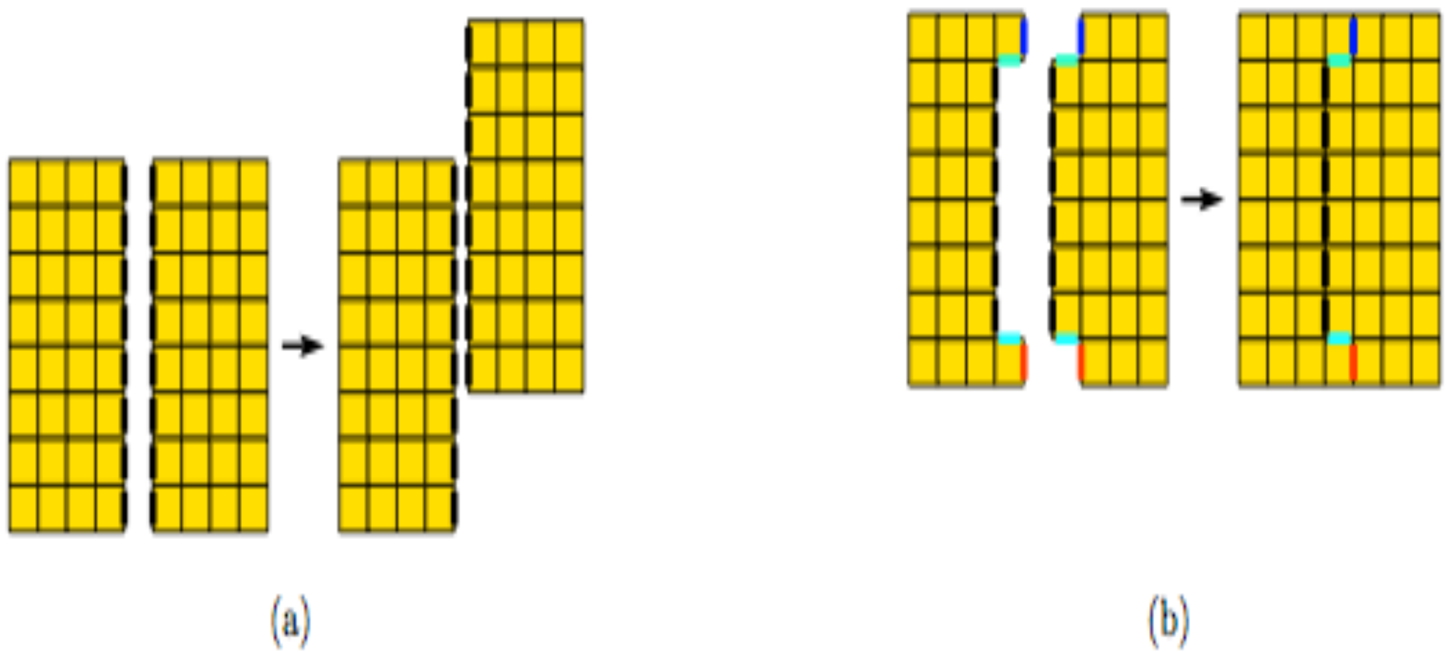


Figure 4: (a) The shifting problem encountered when combining rectangle supertiles. (b) The jigsaw solution: two supertiles that combine uniquely into a fully connected square supertile.

Decompose Algorithm

Algorithm DecomposeVertically (supertile S):

— Here S is a supertile with n rows and m columns; S is not necessarily a rectangle.

1. **Stop vertical partitioning when width is small enough:**
   If $m \leq 3$, DecomposeHorizontally(S) and return.
2. **Find the column along which the supertile is to be partitioned:**
   Let $i := \lfloor (m+1)/2 \rfloor$.
   Divide supertile S along the ith column into a left supertile S_1 and right supertile S_2 such that
   tiles at position $(1, i)$ and (n, i) belong to S_1 and the rest of the ith column belongs to S_2.
3. **Now decompose recursively:**
   DecomposeVertically (S_1)
   DecomposeVertically (S_2)

Decompose Algorithm

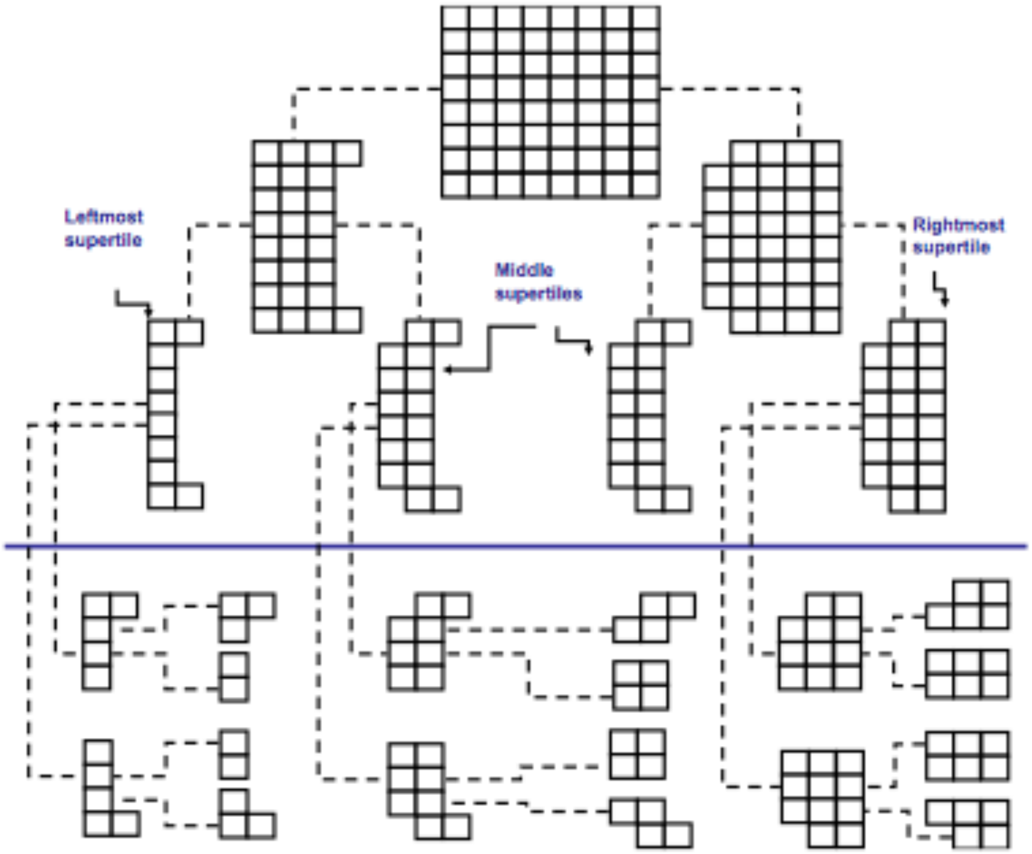


Figure 5: Decomposition tree for **8** × **8** square in the jigsaw technique.

Decompose Algorithm

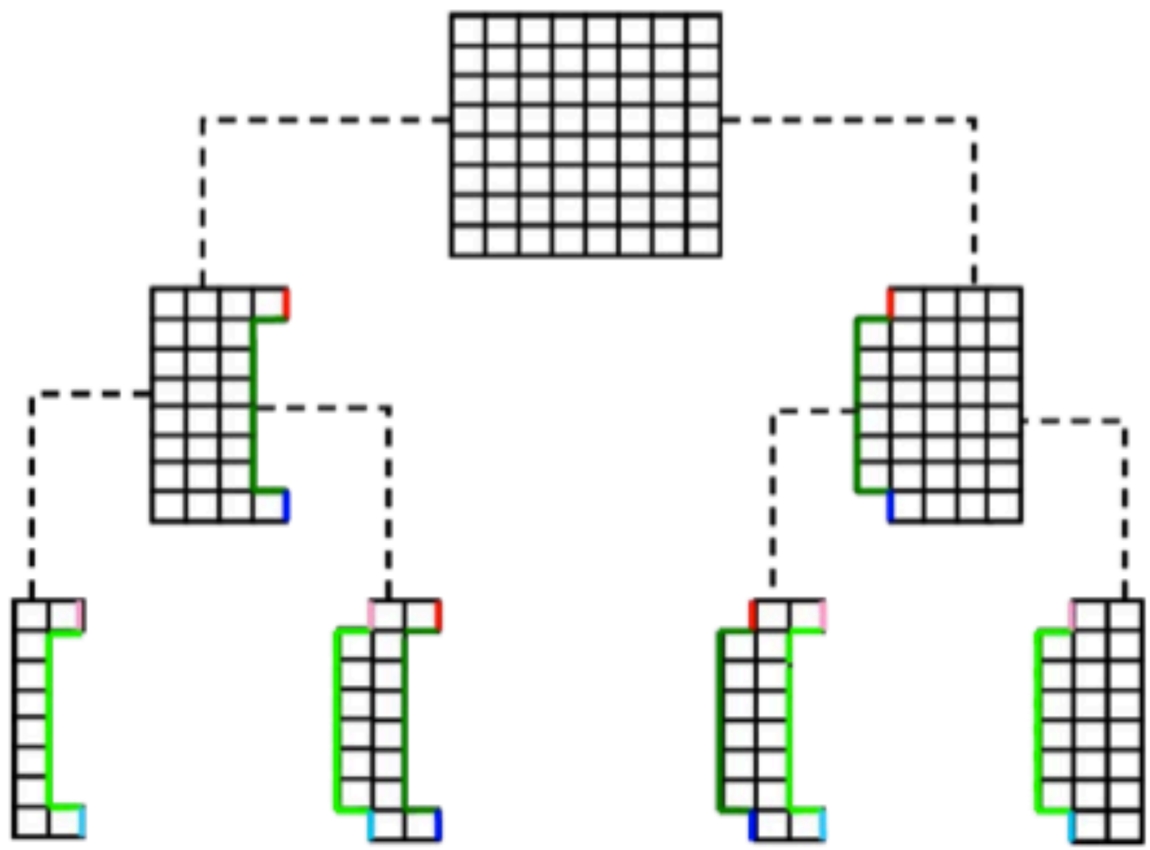


Figure 6: Assigning glues in the first two vertical decompositions of the jigsaw technique.

Flexible Glue Model

- Remove: g(x, y)=0 for x≠y[Aggarwal et al. SODA 2004].
- Assembly of N×N square: the tile complexity of self-assembling N×N squares is $O(\sqrt{\log N})$ under the flexible glue model.

Assembly of k×N Rectangles under Standard Model

- Theorem: the tile complexity of self-assembling a k×N rectangle is O(N^(1/k)+k) for standard model[Aggarwal et al. SODA 2004].
- Trick: a k-digit base-m counter, where m=ceil (N^(1/k)).
- What if N is not a power of m? By initializing.

Assembly of k×N Rectangles under Standard Model

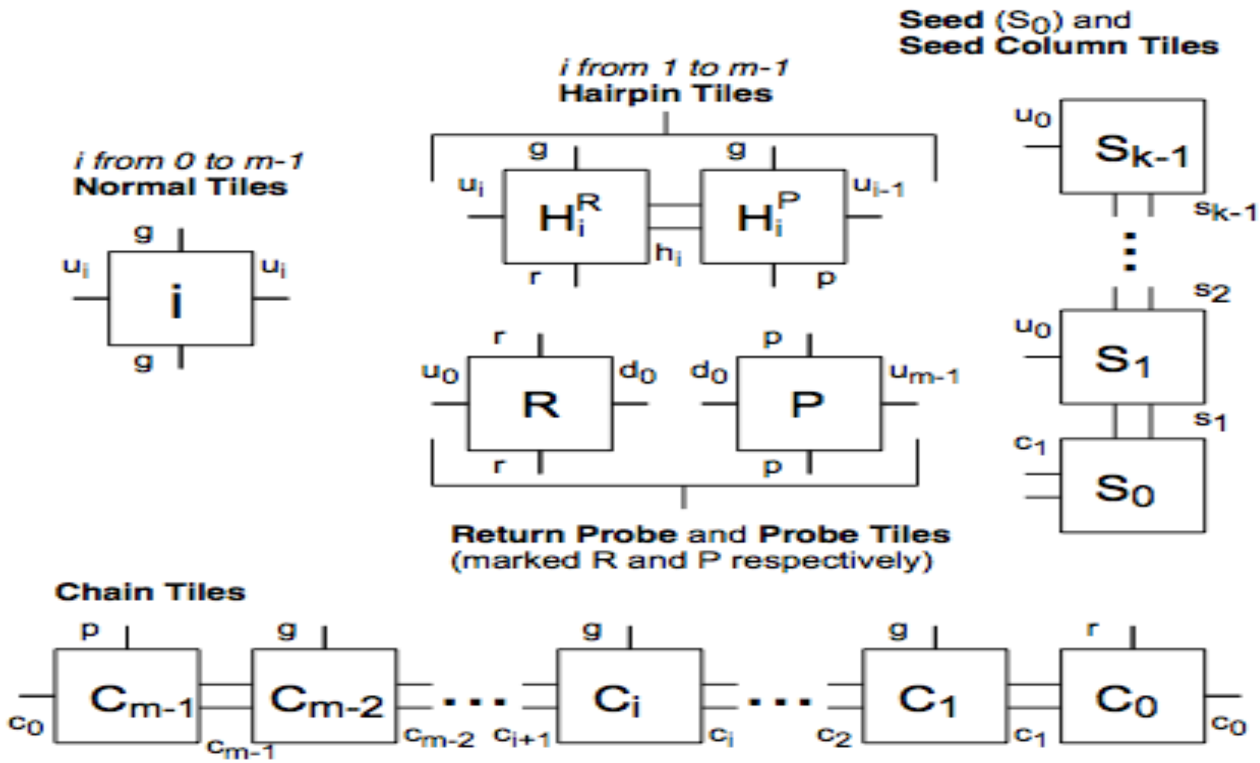


Figure 1: A tile set to assemble a $k \times m^k$ rectangle in $\Theta(k + m)$ tile complexity under the standard assembly model.

Assembly of k×N Rectangles under Standard Model

- C tiles: counting for the 0th bit of the counter.
- H tiles: counting for the other bits of the counter.
- R,P tiles: transfer carry and flip bits back to 0.

Assembly of N×N Square under Flexible Glue Model

- Theorem: the tile complexity of self-assembling N×N squares is $O(\sqrt{\log N})$ under the flexible glue model[Aggarwal et al. SODA 2004].
- Thought 1: the complexity of assembling N×N square is almost the complexity of assembling a (logN)-digit counter.
- Thought 2: how to assemble a (logN)-digit counter by $O(\sqrt{\log N})$ tile types.

Assembly of N×N Square under Flexible Glue Model

- Thought 3: the complexity of assembling a counter is almost the complexity of assembling the first row of the counter.
- Thought 4: how to assemble the first row of a (logN)-digit counter by $O(\sqrt{\log N})$ tile types.
- Target: Assemble the first row of a n-digit counter by $O(\sqrt{n})$ tile types.

Assembly of N×N Square under Flexible Glue Model

- Tile set:

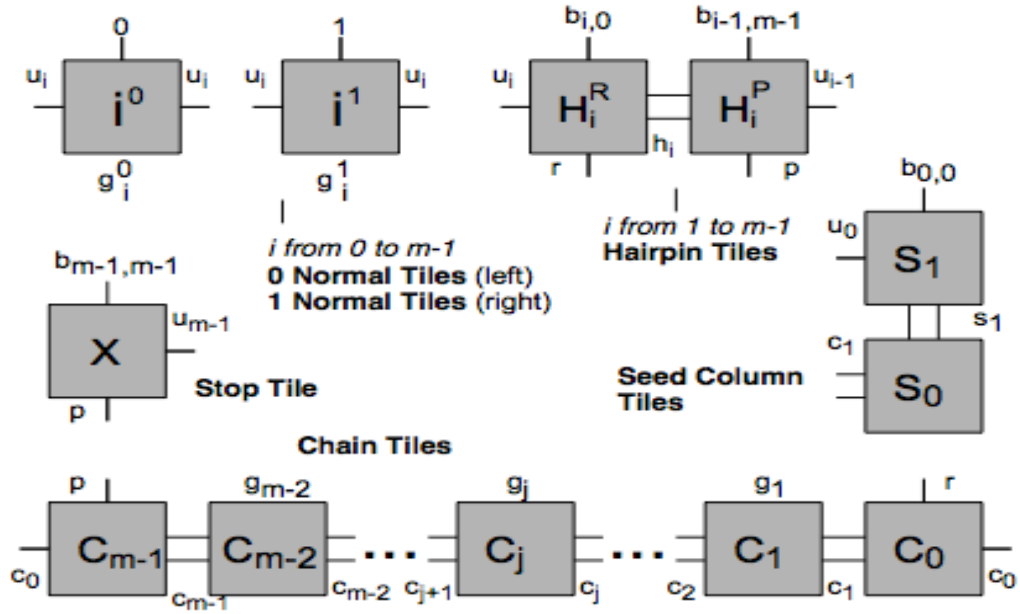


Figure 4: This tile set creates a $2 \times n$ block whose top row represents a given n-bit binary number b. Here b_{ij} is the value of the bit in position $im + j$ in b. The glue function for glues g_i^1 and g_j for i from 0 to $m - 1$ and j from 1 to $m - 2$ is $G(g_i^1, g_j) = b_{ij}$. All other pairs of non-equal glues have strength 0.

Assembly of N×N Square under Flexible Glue Model

- Example:

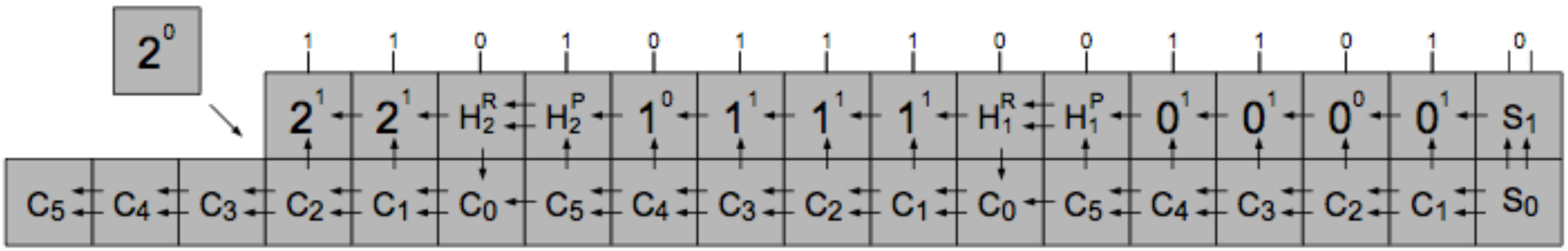


Figure 5: Assembling an arbitrary n-bit binary number in $O(\sqrt{n})$ tiles. Here we show the construction for $n = 36$ and binary number $b = \ldots 0110101110011010$.

Time-dependent Glue Model

- The glue strength between glue x, y is a function of time t: $f_{x,y}(t)$[Sahu et al. DNA 2005].

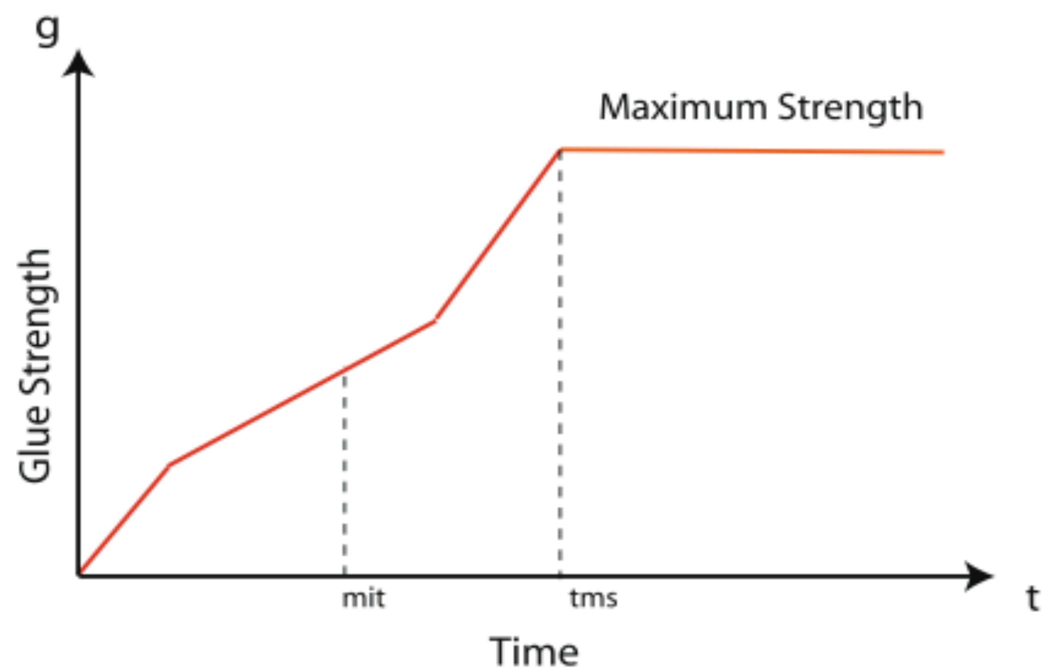


Fig. 1 Figure illustrates the concept of time-dependent glue strength, minimum interaction time, and time for maximum strength

Time-dependent Glue Model

- Minimum interaction time.
- Complexity result: the tile complexity of assembling a $k \times N$ rectangle is $O(\log N / \log\log N)$ under time-dependent glue model, where $k < \log N / (\log\log N - \log\log\log N)$[Sahu et al. DNA 2005].
- Trick: assemble a $j \times N$ rectangle where $j > k$. Disassemble the upper $(j-k)$ rows.