

Minimum Cut

deterministic algorithms

- Max-flow
- Gabow

Min-cut implementation

- data structure for contractions
- alternative view—permutations.
- deterministic leaf algo

Recursion:

$$\begin{aligned} p_{k+1} &= p_k - \frac{1}{4}p_k^2 \\ q_k &= 4/p_k + 1 \\ q_{k+1} &= q_k + 1 + 1/q_k \end{aligned}$$

MST

Background

- kruskal
- boruvka
- verification

sampling theorem:

- Heavy edges
- pick F with probability p
- get n/p F -heavy edges

Recursive algorithm without boruvka:

$$T(m, n) = T(m/2, n) + O(m) + T(2n, n) = O(m + n \log n)$$

(sloppy on expectation)

Recursive algorithm with 3 boruvka steps:

$$\begin{aligned} T(m, n) &= T(m/2, n/8) + c_1(m + n) + T(n/4, n/8) \\ &\leq c(m/2 + n/8) + c_1(m + n) + c(n/4 + n/8) \\ &= (c/2 + c_1)m + (c/8 + c_1 + c/4 + c/8)n \\ &= (c/2 + c_1)(m + n) \end{aligned}$$

so set $c = 2c_1$

Notes:

- Chazelle $m \log \alpha(m, n)$ via relaxed heap
- open questions.

Samples for Cuts

- cut counting
- Reliability
- Gabow's algorithm
- Sampling