We first prove a cutting lemma that is a 3-dimensional variant of a lemma shown in [Tya89]. In this section, we use the term box to refer to a rectilinear parallelepiped rather than an optical box.

**Lemma 1** Let $C = (V, W)$ compute $f$ as described above. Let us assume that no input port reads more than $n/486$ input bits. There exist two sets of input ports $I_1$ and $I_2$ such that $I_1$ and $I_2$ each read at least $n/486$ input bits. Let $C_{I_1}$ and $C_{I_2}$ denote the smallest boxes containing $I_1$ and $I_2$ respectively. Let $s_1$ ($s_2$) be the surface area of $C_{I_1}$ ($C_{I_2}$). Let $d_x$, $d_y$ and $d_z$ be the distance between $C_{I_1}$ and $C_{I_2}$ along $x$, $y$ and $z$ axes respectively. Then one of the following statements holds true.

1. $d_x \geq \min(\sqrt{s_1/6}, \sqrt{s_2/6})$.
2. $d_y \geq \min(\sqrt{s_1/6}, \sqrt{s_2/6})$.
3. $d_z \geq \min(\sqrt{s_1/6}, \sqrt{s_2/6})$.

**Proof Sketch:** Since no input port reads more than $n/486$ input bits, we can find a vertical plane orthogonal to $x$–axis $V_x$ bisecting the chip into two