halves such that each half reads at least \( n/3 \) input bits. The argument for it is similar to the argument in Ullman ([Ull84], page 49). Let us concentrate on the right-hand side of \( V_x \). Once again, we can find a vertical plane orthogonal to the \( y \)-axis \( V_y \) that cuts the right-hand side of \( V_x \) into two parts such that each part reads at least \( n/9 \) input bits. The left-hand side of \( V_x \) is also cut into two parts by the plane \( V_y \). At least one of these parts reads \( n/6 \) input bits. Without loss of generality (WLOG), let the south-west quadrant \( SW \) read at least \( n/6 \) input bits as shown in Figure 1. Let us further cut this south-west part with a plane orthogonal to \( z \)-axis \( H_z \) such that both the halves of \( SW \) read at least \( n/18 \) input bits. This plane also cuts the the north-east part \( NE \) into two parts. At least one of these parts reads at least \( n/18 \) input bits. WLOG, let the upper part \( NE_u \) read at least \( n/18 \) input bits. In the following, we will consider \( NE_u \) and the lower part of \( SW \), \( SW_l \). Let \( C_{I_1} \) and \( C_{I_2} \) be the smallest boxes containing the input ports in the \( SW_l \) and \( NE_u \) quadrants respectively, as shown in Figure 2.

Now cut both \( C_{I_1} \) and \( C_{I_2} \) as follows to create 8 boxes each reading at least \( n/486 \) input bits. First use a vertical plane \( V'_z \) to cut \( C_{I_1} \) into two parts each reading at least \( n/54 \) input bits. Then cut each of these parts with a plane orthogonal to \( y \) axis to get four parts each reading at least \( n/162 \) input bits. Now use a plane orthogonal to \( z \)-axis to cut each of these

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Figure 2: Further Cutting in the Cutting Lemma