CPS 140 **Exam 2 Solutions** **Spring 1995**

Dr. Rodger

1. (3 pts) Let $L_3 = \{a^n b^{2n} c^p \mid n > 0, p > 0, p < n\}$. L_3 is not a context-free language. Give two context-free languages L_1 and L_2 such that $L_1 \cap L_2 = L_3$.

   $L_1 = \{a^n b^{2n} c^m \mid n > 0, m > 0\}$, $L_2 = \{a^n b^m c^p \mid n > 0, m > 0, p > 0, p < n\}$

2. (8 pts) Consider the following languages. Write "CFL" if a language is a context free language, and write "NOT" if it is not a context free language.

   (a) **L**$=\{a^n b^m c^{2n} d^{2n} | n > 0, m > 0\}$ **NOT**

   (b) **L**$=\{(ab)^n c^n d^m | n > 0, m > 0, n \neq m\}$ **NOT**

   (c) **L**$=\{a^{2n} b^m c^n | n > 0, m \geq 0$, n **is even and** m **is odd** $\}$ **CFL**

   (d) **L**$=\{a^n b^m c^p | n > 0, m > 0, p > 0, m <= n + p\}$ **CFL**

3. (3 pts) The following grammar is not LR(1). What type of conflict is encountered: Shift/Shift, Reduce/Reduce, or Shift/Reduce? Explain.

   S → Aac | Bab
   A → b
   B → b

   A reduce/reduce conflict is encountered with the first symbol b, the parser doesn't know which rule to reduce by, A → b or B → b, since a is in the FOLLOW(A) and in the FOLLOW(B).

4. (2 pts) For any context-free language L, there exists a Turing Machine M such that L=L(M). (True or False?)

   TRUE

5. (2 pts) When was the Turing machine invented: 1830's, 1860's, 1930's, or 1960's?

   1930's

6. (3 pts) The following relates to a pushdown automaton M. Explain what this definition means.

   $$L(M)=\{w \in \Sigma^* | (q_0, w, z) \overset{*}{\vdash} (p, \lambda, \lambda)\}$$

   This is the definition of acceptance by empty stack. The language of the machine is the set of strings such that if you start in state q_0 with w on the tape and w is in the language of L, then after processing all the symbols in w, the machine is in some state p with the stack empty. (If w is not in L, after processing all the symbols in w, the stack will not be empty.)

7. (8 pts) A deterministic Turing machine M is defined by M=(Q,Σ,Γ,δ,q_0,B,F). What do each of these symbols mean?

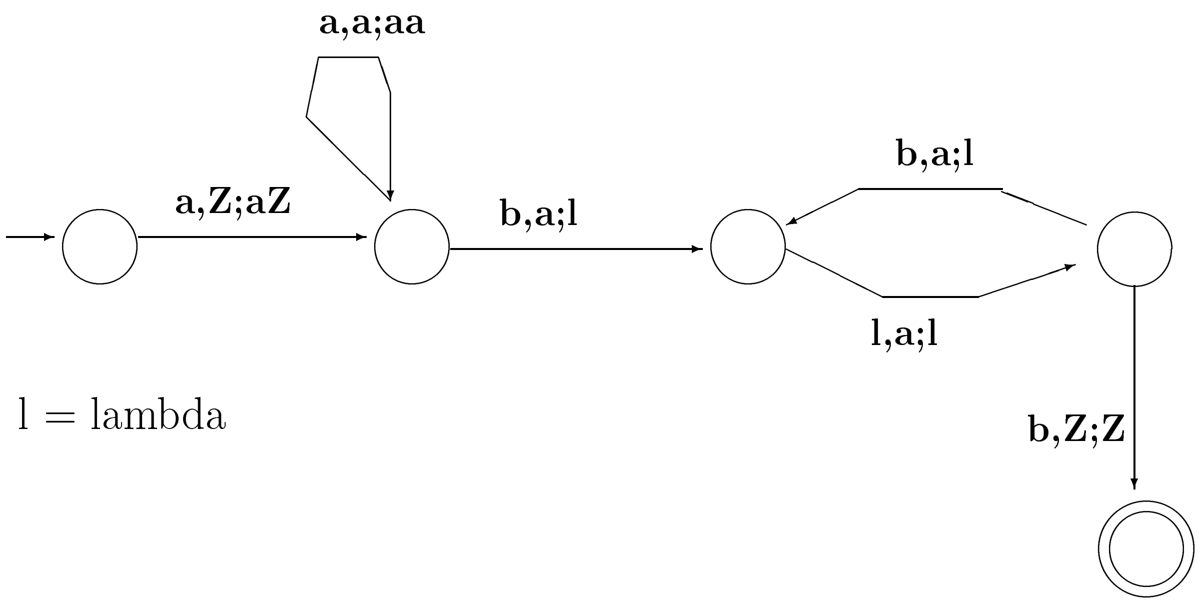
   (a) Q is **finite set of states**
   (b) Σ is **the input alphabet.**
   (c) Γ is **tape alphabet**
   (d) δ:Q$\times\Gamma \rightarrow Q \times \Gamma \times \{L, R\}$
   (e) q_0 is **the start state**
   (f) B is **the symbol for blank**
   (g) F is **the set of final states**

8. (12 pts) Consider $L = \{a^{2n}b^{n+1} \mid n > 0\}$. Draw the transition diagram for a nondeterministic pushdown automaton M that accepts L by final state. (Remember to identify the start state by an arrow and final states by double circles. Format of labels are $a, b; cd$ where a is the symbol on the tape, b is the symbol on top of the stack that is popped, and cd are pushed onto the stack (with c on top of d). Z is on top of the stack when M starts.).

   (a) First list the three smallest strings in L.

   aabb, aaaabbb, aaaaaabbbb

   (b) Now draw the transition diagram.



1 = lambda

9. (15 pts) Consider the following grammar (DO NOT change the grammar):

$$
\begin{aligned}
&S \to ABS \mid c \mid \lambda \\
&A \to a \\
&B \to bSb \mid \lambda
\end{aligned}
$$

(a) Calculate FIRST and FOLLOW for the variables in the grammar.

	FIRST	FOLLOW
S	**a, c,** λ	**b,** $
A	**a**	**b, c, a,** $
B	**b,** λ	**a, c, b,** $

(b) Calculate all entries in the LL(1) Parse Table. If there are multiple rules for an entry, write in all the rules.

	a	b	c	$
S	**ABS**	λ	**c**	λ
A	**a**			
B	λ	λ, bSb	λ	λ

(c) Is the grammar LL(1)? Explain.

No, there are two rules when B is on the stack and b is the lookahead.

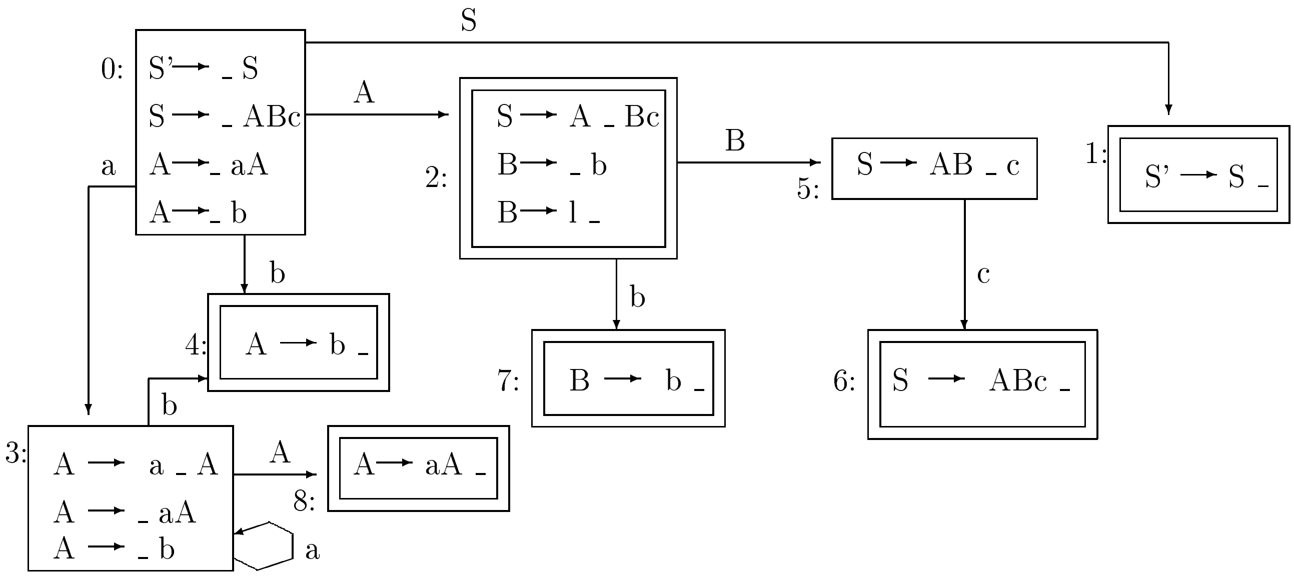
10. (20 pts) Construct the LR parsing table for the following grammar. A new start symbol S' and production have already been added to the grammar.

1) S' $\to$ S	3) A $\to$ aA	5) B $\to$ λ
2) S $\to$ ABc	4) A $\to$ b	6) B $\to$ b

(a) Calculate the FIRST and FOLLOW sets of variables.

	FIRST	FOLLOW
S	a,b	\$
A	a,b	b,c
B	b,λ	c

(b) Construct the transition diagram of the DFA that models the stack. Number the states, show marked productions, and identify final states by two circles.



(c) Construct the LR parse table that corresponds to the transition diagram drawn in part b. (Note: all the rows and columns given may not be needed. If there are multiple items for an entry, put both.)

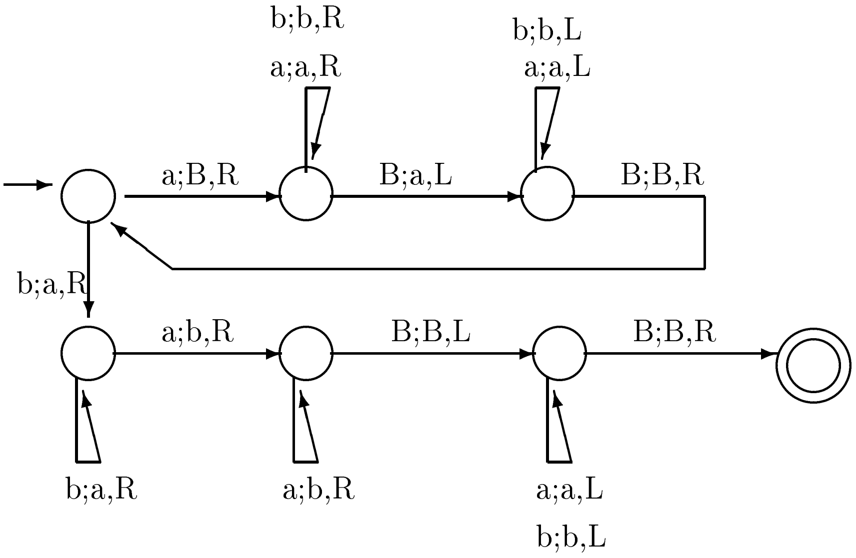
	a	b	c	$	S	A	B
0	s3	s4			1	2	
1				acc			
2		s7	r5				5
3	s3	s4				8	
4		r4	r4				
5			s6				
6				r2			
7			r6				
8		r3	r3				

(d) Is the grammar LR(1)? Explain.

yes, there are no conflicts in the table.

11. (12 pts) Design a standard Turing machine for the function f($a^n b^m$) $= a^m b^n$ (assume $n > 0, m > 0$) by drawing a **transition diagram**. (Remember to identify the start state by an arrow and final states by double circles. Format of labels are $a; b, M$ where a is the symbol read on the tape, b is the symbol written on the tape, and M is the movement of the tape head (either left(L) or right(R)). B is the symbol for a blank.).

For example, if the tape head starts on the leftmost symbol of $aabbbb$ then when the Turing machine halts the tape head will be pointing to the leftmost symbol of $aaaabb$. You may assume the input is in the correct format.



What is the running time of your Turing machine?

O($(n+m)^2$)

12. (12 pts) **Pumping Lemma for CFL's** Let L be any infinite CFL. Then there is a constant m depending only on L, such that for every string w in L, with $|w| \geq m$, we may partition $w = uvxyz$ such that:

$|vxy| \leq m$, (limit on size of substring)
$|vy| \geq 1$, (v and y not both empty)
For all $i \geq 0$, $uv^ixy^iz \in$L

Prove that L=$\{a^n b^{2n} c^p | p < n, p > 0, n > 0\}$ is not a context-free language.

(a) Assume: **L is a context free language.**

(b) Choose $w = a^m b^{2m} c^{m-1}$

(c) Explain why v cannot contain two or more distinct symbols.
If v contains a's and b's, then $uv^2xy^2z \notin$L since there will be b's before a's.

(d) Prove the case when $v = a^{t_1}$ (is a string of a's)

$v = a^{t_1}$**, then** $y = a^{t_2}$ **or** b^{t_3} $(|vxy| \leq m)$

If $y = a^{t_2}$**, then** $uv^2xy^2z = a^{m+t_1+t_2}b^{2m}c^{m-1} \notin L$ **since** $t_1 + t_2 > 0$**, 2 times the number of** a**'s is not equal to the number of** b**'s.**

If $y = b^{t_3}$**, then** $uv^0xy^0z = a^{m-t_1}b^{2m-t_3}c^{m-1} \notin L$ **since** $t_1 + t_3 > 0$**, either 2 times the number of** a**'s is not equal to the number of** b**'s OR the number of** a**'s is not greater than the number of** c**'s.**

(e) Prove the case when $v = b^{t_1}$ (is a string of b's)

$v = b^{t_1}$**, then** $y = b^{t_2}$ **or** c^{t_3} $(|vxy| \leq m)$

If $y = b^{t_2}$**, then** $uv^2xy^2z = a^{m}b^{2m+t_1+t_2}c^{m-1} \notin L$ **since** $t_1 + t_2 > 0$**, 2 times the number of** a**'s is not equal to the number of** b**'s.**

If $y = c^{t_3}$**, then** $uv^2xy^2z = a^{m}b^{2m+t_1}c^{m-1+t_3} \notin L$ **since** $t_1 + t_3 > 0$**, either 2 times the number of** a**'s is not equal to the number of** b**'s OR the number of** a**'s is not greater than the number of** c**'s.**

(f) Assuming all of the cases for v can find contradictions, explain what this means.

Thus, there is no breakdown of w **into** $uvxyz$ **such that** $|vy| \geq 1$**,** $|vxy| \leq m$ **and for all** $i \geq 0$**,** uv^ixy^iz **is in** L**. Contradiction, thus,** L **is not a CFL. Q.E.D.**