CPS 140 **Exam 2** **Spring 1998**

Dr. Rodger

1. (16 pts) Answer TRUE or FALSE to the following statements.

   (a) The pumping lemma for context-free languages can be used to prove a language is context-free. (TRUE or FALSE?)

   (b) If L is a regular language, then there exists a NTM (nondeterministic TM) M such that L=L(M). (TRUE or FALSE?)

   (c) If M is a NPDA, then there exists a DTM (deterministic TM) M' such that L(M) = L(M'). (TRUE or FALSE?)

   (d) If M is a DTM with $|\Sigma|=1$, then there exists a NPDA M' such that L(M)=L(M'). (TRUE or FALSE?)

   (e) L=$\{a^n b^m c^{2n} \mid n > 0, m > 0\}$ is a CFL. (TRUE or FALSE?)

   (f) L=$\{a^{2n} b^m c^p \mid n > 0, m > n, m > p\}$ is a CFL. (TRUE or FALSE?)

   (g) L=$\{a^n b^m c^n d^p \mid 0 < n < 30, p > m > n\}$ is a CFL. (TRUE or FALSE?)

   (h) L=$\{w \mid n_a(w) > n_b(w)$ and $n_a(w) mod 3 = 0\}$ is a CFL. (TRUE or FALSE?)

2. (3 pts) Both the LL and LR parser process the input string from left to right. Briefly explain the main difference between these methods.

3. (3 pts)

   Consider the following LR(1) grammar.

   S → ABc | Bd
   A → ab
   B → ab

   In the DFA of marked rules, there is a state with the two marked rules:

   A → ab_
   B → ab_

   Explain why this state does NOT have a reduce/reduce conflict.

4. (12 pts) Consider $L = \{a^n b^m \mid 0 < n \leq m \leq 2n, \}$. Draw the transition diagram for a nondeterministic pushdown automaton M that accepts L by final state. (Remember to identify the start state by an arrow and final states by double circles. Format of labels are $a, b; cd$ where a is the symbol on the tape, b is the symbol on top of the stack that is popped, and cd are pushed onto the stack (with c on top of d). Z is on top of the stack when M starts.).

(a) First list 4 strings in L.

(b) Now draw the transition diagram.

5. (12 pts) Consider the following grammar (DO NOT change the grammar):

$$S \rightarrow ASB \mid aA$$
$$A \rightarrow AB \mid \epsilon$$
$$B \rightarrow bBc \mid \epsilon$$

This grammar is not LL(1). For this problem you will go ahead and construct the LL parser showing all the conflicts.

(a) Calculate FIRST and FOLLOW for the variables in the grammar.

	FIRST	FOLLOW
S		
A		
B		

(b) Calculate all entries in the LL(1) Parse Table. If there are multiple rules for an entry, write in all the rules.

	a	b	c	$
S				
A				
B				

(c) This grammar is LL(k) for what value of k? Explain.

6. (20 pts) Construct the LR parsing table for the following grammar (DO NOT change the grammar.) A new start symbol S' and production have already been added to the grammar.

1) S' $\rightarrow$ S 2) S $\rightarrow$ BD 3) B $\rightarrow$ bB
4) B $\rightarrow$ c 5) D $\rightarrow$ Dd 6) D $\rightarrow \epsilon$

(a) Calculate the FIRST and FOLLOW sets of variables.

	FIRST	FOLLOW
S		
B		
D		

(b) Construct the transition diagram of the DFA that models the stack. Number the states, show marked productions, and identify final states by two circles.

(c) Construct the LR parse table that corresponds to the transition diagram drawn in part b. (Note: all the rows and columns given may not be needed. If there are multiple items for an entry, put both.)

0									
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									

7. (10 pts) **Pumping Lemma for CFL's** Let L be any infinite CFL. Then there is a constant m depending only on L, such that for every string w in L, with $|w| \geq m$, we may partition $w = uvxyz$ such that:

$|vxy| \leq m$, (limit on size of substring)
$|vy| \geq 1$, (v and y not both empty)
For all $i \geq 0$, $uv^ixy^iz \in$L

Prove that L=$\{a^nb^qc^nd^p | n > 0, q > 0, p > q\}$ is not a context-free language.

Assume L is a context-free language.

(a) Choose $w =$

(b) Explain why v cannot contain two or more distinct symbols.

(c) Prove the case when $v = a^{t_1}$ (is a string of a's)

(d) Prove the case when $v = d^{t_1}$ (is a string of d's)

8. (12 pts) Construct a TM (using a transition diagram) for the following function. $f(w) = w'$, where $\Sigma = \{a, b, c\}$, $w \in \Sigma^+$, and w' is w with every other a (starting with the first a) replaced with a c.

For example, $f(aabaacaba) = acbaccabc$

In drawing the transition diagram, remember to identify the start state by an arrow and final states by double circles. Format of labels are $a; b, R$ where a is the symbol read on the tape, b is the symbol written to the tape and R is the direction moved (you can use L, R or S for directions.) Make sure the tape head is pointing to the leftmost symbol of the output.

Assume $|w| = n$. What is the worst case running time (big-Oh) of your Turing machine?

BUILDING BLOCK NOTATION

(a) s - start

(b) R - move right one cell

(c) L - move left one cell

(d) x - write x (and don't move)

(e) R_a - move right until you see an a (always moves at least one cell, ignores the symbol on the cell the tape head starts at)

(f) L_a - move left until you see an a

(g) $R_{\neg a}$ - move right until you see anything that is not an a

(h) $L_{\neg a}$ - move left until you see anything that is not an a

(i) $R_{a|b}$ - move right until you see an a or a b

(j) $L_{a|b}$ - move left until you see an a or a b

(k) h - halt in a final state

(l) $\overset{a,b}{\rightarrow}\} \overset{w}{\rightarrow}$

If the current symbol is a or b, let w represent the current symbol.

9. (12) Construct a TM (using building blocks) for the following function. $f(x\#y) = T$ if $x < y$ and is equal to F otherwise, where x and y are unary numbers. You can use any of the building block notation on the previous page (note that the | symbol (or) has been added as rules (i) and (j)).

   For example, $f(111\#11)$ writes F to the tape, and $f(11\#1111)$ writes T to the tape. The answer must be surrounded by blanks. It is ok to have other stuff still on the tape when the Turing machine halts.

   Assume $|x| = n$ and $|y| = m$. What is the worst case running time (big-Oh) of your Turing machine?