Mechanism Design for Scheduling with Uncertain Execution Time.

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The Queen wants a painting for her palace. Every day she decides which painters will draw.

Goal: minimize $E[\text{painting time}]$

*Creativity is unpredictable! Painters don’t know how much time it’s going to take them and need incentives to draw!*
Time painter $i$ needs to finish the job ~ distribution $f_i$

painter $i$ knows $f_i$ but not $t_i$ and $f_i$

Monotone hazard rates: the more time a painter takes the less likely is he to finish at the next time step
CS Applications

We want to solve a SAT instance, using multiple cloud providers.

The machines don’t know how much time it will take them.

For SAT there are some good euristics so the probability of finding a solution in the beginning is rather high, but if these don’t work then it might take forever... (MHR assumption makes sense)
Hazard Rate:
Probability a painter finishes the painting at time $t$ given that he hasn’t finished it until time $t-1$

$$h_i(t) = \frac{P(T_i = t)}{(1 - P[T_i < t])}$$

Monotone hazard rate assumption:
the more time a painter takes
the less likely is he to finish at the next time step
The greedy algorithm is optimal!

Objective: Minimize the $E[\text{sum of processing times}]$

**Greedy=OPT:** assign at each time step the job to the machine with maximum hazard rate i.e. the machine more likely to finish!

To prove this we need: Monotone hazard rates assumption

- “Off with their heads!”
Objective: Minimize the $E[\text{sum of processing times}]$

**OPT**: assign at each time step the job to the machine with maximum hazard rate

```
h(1)  h(2)  h(3)  h(4)  h(5)  ...
0.9  0.8  0.5  0.1  0.1
0.7  0.4  0.3  0.3  0.3
0.6  0.2  0.1  0.1  0.1
```

**Input**

```
0.9  0.8  0.5  0.1  0.1
0.7  0.4  0.3  0.3  0.3
0.6  0.2  0.1  0.1  0.1
```

**OPT**

```
0.9  0.8  0.7  0.6  0.5  0.4  0.3  0.3
```
Consistency Property

If we remove one player e.g. to get OPT for the rest of the players we just need to remove the player from the schedule.

```
Input
h(1) h(2) h(3) h(4) h(5)
0.9  0.8  0.5  0.1  0.1
0.7  0.4  0.3  0.3  0.3
0.6  0.2  0.1  0.1  0.1

OPT
0.9  0.8 0.5      0.7 0.4  0.3  0.30.6
0.7  0.6  0.4  0.3  0.3
0.4  0.3  0.3
```
We focus on **direct revelation mechanisms**

**Input:** true types of the players here: the distributions $f_i$

**Output:** allocation (which machine processes at each time step) and payments

We have a **revelation principle**
The fastest machine (lowest bid) $t_i$ wins and gets paid the 2nd lowest bid.

The Vickrey mechanism:
- The highest bid $v_i$ wins and pays the 2nd highest bid.
- The Vickrey mechanism is truthful and minimizes the sum of processing times.

Auctions:
- Item for sale: job to process

Scheduling
A bad mechanism: Expected Groves

\[ \text{Groves payment} = -(E + \text{sum of the Expected times of the other players}) \]

not ex-post truthful!!
After completing the task we have the realized running times of the players

Groves payment $= -(\text{OPT + Groves Realized}) - \text{"sum of the realized times of the other players"}$
Solution Concept: Ex-post equilibrium

If the other players are telling the truth, then the best thing for me to do is to tell the truth, for any private information the players might.

Dominant $\preceq$ Ex-post $\preceq$ Bayes Nash
Valuations are interdependent

The valuation of a player depends on whether another player has already finished before him.
If we want our mechanism to have an $h_i$ (types of the other players) part (useful for getting properties like IR, etc)

We have to consider the situation when the player who finishes isn’t there.
“How much does the player who finish contribute to the social wellfare?”

What if the player who finishes wasn’t there?
Vickrey Variations

\[ T_N := \text{how long it takes a group N to finish the task (random variable)} \]

\[ r_N := \text{realized value of } T_N \]

\[ \text{payment}_i = E[T_N - T_i] + E[T_{N \setminus \{i\}}] \]

\[ \text{payment}_i = -(r_N - r_i) + 0 \]

\[ \text{payment}_i = -(r_N - r_i) + E[T_{N \setminus \{i\}}] \]

\[ \text{payment}_i = -(r_N - r_i) + (r_N - r_i) + E[T_{N \setminus \{i\}} - r_{N \setminus \{i\}} | T_{N \setminus \{i\}} \geq r_{N \setminus \{i\}}] \]

This rewritting uses the consistency property!
### Properties of different Mechanisms

<table>
<thead>
<tr>
<th></th>
<th>efficient</th>
<th>truthful in dominant strategies</th>
<th>ex-post truthful</th>
<th>IR</th>
<th>no incentive to miscompute</th>
<th>payment 0 if fail</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Clarke in Expectation</strong></td>
<td>✔</td>
<td>✗</td>
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<td><strong>Pure Realized Groves (PRG)</strong></td>
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<td><strong>Clarke h partially in Expectation</strong></td>
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No incentive to miscompute

Maybe the painters reported their true distributions but in the end decided it is to their best interest to take a break instead of painting!

If no realized values are used then players can just sit and compute nothing!
“The payments align the incentives of the players with the objective of the mechanism”

Groves payment = \(- (\text{Utility of selfish player}) + \text{valuation} + \text{Groves payment}\)

 Thus no incentive to lie or miscompute!
No incentive to lie or miscompute in ChpE (Clarke h partially in Expectation)
Proof is more involved. Idea:

\[
\text{payment}_i = -(r_N - r_i) + (r_N - r_i) + E[T_{N\{i\}} - r_{N\{i\}} | T_{N\{i\}} \geq r_{N\{i\}}]
\]

When player i finishes the task he determines which part is taken in expectation and which not. Suppose that he had even more power and could “cut” h at any point then he could still not affect the Expectation of h by miscomputing!
“Would you tell me, please, which way I ought to go from here?”

“That depends a good deal on where you want to get to.”

“I don't much care where –”

“Then it doesn't matter which way you go.”  Lewis Carol, Alice in Wonderland