Mechanism Design for Scheduling with Uncertain Execution Time.

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Joint work with

• The setting

Goal of mechanism designer:
minimize \( E [\text{sum of painting times}] \)
Every day she decides which painters will draw.

Time painter \( i \) needs to finish the job \( \sim \) distribution \( f_i \)
painter \( I \) knows the distribution \( f_i \) (this is his type) from which his painting time is drawn but not his painting time \( t_i \) or the distributions of the other players \( f_j \).

Players are selfish
want to maximize their utility,
which is:
\[ E [\text{payment} - \text{time spent painting}] \]

Monotone hazard rate assumption:
The probability a painter finishes the painting at time \( t \) given that he hasn’t finished it until time \( t-1 \) is non-increasing.

We want a mechanism where the players have no incentive to misreport their types or miscompute.

• The efficient solution

Greedy=OPT: assign at each time step the job to the machine with maximum hazard rate, i.e. the machine most likely to finish!
To prove this we need: Monotone hazard rates assumption

OPT satisfies the Consistency Property:
"If we remove one player, to get OPT for the rest of the players we just need to remove the player from the schedule."

• Expected Clarke isn’t truthful

The player who is most likely to finish at the first time-step has an incentive to over-report his probability of finishing at the first step.

• Groves Realized is ex-post truthful

After completing the task we have the realized running times

Groves payment \( = \frac{1}{N}\sum_{j \neq i} \text{subject to } h \text{(types of the other players) part} \)

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“How much does the player who finish
contribute to the social welfare?”

OPT

What if the player who finishes wasn’t there?

REALIZED TIMES OF THE OTHER PLAYERS

FINISH TIME

h part of the mechanism


• Solution concept: ex-post equilibrium

Valuations are interdependent: a player’s utility is affected by the other players’ true distributions because those will affect the probability that she gets to run.

Ex-post equilibrium If the other players are telling the truth, then the best thing for me to do is to tell the truth, for any private information the players might have.

Dominant \( \subseteq \) Ex-post \( \subseteq \) Bayes Nash

• Vickrey Variations

\( h(\text{types of the other players}) \) part

T \( = \) how long it takes a group \( N \) to finish the task (random variable)
r \( = \) realized value of \( T \)

\( \text{OPT satisfies the: Consistency Property} \)

Expected Clarke (CE)

Pure Realized Groves (PRG)

Expected Clarke

(CheE) Clarke h partially in Expectation

\( h(\cdot) \)

This rewriting uses the consistency property!

The payments align the incentives of the players with
the objective of

Groves payment

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• Properties of different Mechanisms

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<tr>
<th>Mechanism</th>
<th>efficient</th>
<th>truthful in dominant strategies</th>
<th>ex-post truthful</th>
<th>IR</th>
<th>no incentive to miscompute</th>
<th>payment 0 if fail</th>
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<tbody>
<tr>
<td>Expected Clarke</td>
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