Solving Problems Recursively

- Recursion is an indispensable tool in a programmer’s toolkit
  - Allows many complex problems to be solved simply
  - Elegance and understanding in code often leads to better programs: easier to modify, extend, verify (and sometimes more efficient!! See expotest.cpp)
  - Sometimes recursion isn’t appropriate, when it’s bad it can be very bad—every tool requires knowledge and experience in how to use it
- The basic idea is to get help solving a problem from coworkers (clones) who work and act like you do
  - Ask clone to solve a simpler but similar problem
  - Use clone’s result to put together your answer
- Need both concepts: call on the clone and use the result

Print words entered, but backwards

- Can use a vector, store all the words and print in reverse order
  - The vector is probably the best approach, but recursion works too
  
```cpp
void PrintReversed()
{
    string word;
    if (cin >> word) {  // reading succeeded?
        PrintReversed();  // print the rest reversed
        cout << word << endl; // then print the word
    }
}
```
```cpp
int main()
{
    PrintReversed();
}
```
- The function PrintReversed reads a word, prints the word only after the clones finish printing in reverse order
  - Each clone has its own version of the code, its own word variable

Exponentiation

- Computing $x^n$ means multiplying $n$ numbers (or does it?)
  - What’s the easiest value of $n$ to compute $x^n$?
  - If you want to multiply only once, what can you ask a clone?

```cpp
double Power(double x, int n)
// post: returns $x^n$
{
    if (n == 0)
    {
        return 1.0;
    }
    double semi = Power(x, n/2);
    if (n % 2 == 0)
    {
        return semi*semi;
    }
    return x * semi * semi;
}
```
- What about an iterative version?

Faster exponentiation

- How many recursive calls are made to compute $2^{1024}$?
  - How many multiplies on each call? Is this better?

```cpp
double Power(double x, int n)
// post: returns $x^n$
{
    if (n == 0) {
        return 1.0;
    }
    double semi = Power(x, n/2);
    if (n % 2 == 0) {
        return semi*semi;
    }
    return x * semi * semi;
}
```
- What about an iterative version of this function?
Recursive and Iterative log powers

- In the program expotest.cpp we calculate $x^n$ using log($n$) multiplications (basically). We do this both iteratively and recursively using BigInt variables
  - We saw the iterative code in Chapter 5 with doubles
  - BigInt has overloaded operators so these values work like ints and doubles
  - We use the CTimer class to time the difference in using these functions (ctimer.h)

- The recursive version is faster, sometimes much faster
  - Using doubles we wouldn’t notice a difference
  - Artifact of algorithm? Can we “fix the problem”?
  - Natural version of both in programs, optimizing tough.

Back to Recursion

- Recursive functions have two key attributes
  - There is a base case, sometimes called the exit case, which does not make a recursive call
    - See print reversed, exponentiation, factorial for examples
  - All other cases make a recursive call, with some parameter or other measure that decreases or moves towards the base case
    - Ensure that sequence of calls eventually reaches the base case
    - “Measure” can be tricky, but usually it’s straightforward

- Example: sequential search in a vector
  - If first element is search key, done and return
  - Otherwise look in the “rest of the vector”
  - How can we recurse on “rest of vector”?

Classic examples of recursion

- For some reason, computer science uses these examples:
  - Factorial: we can use a loop or recursion (see facttest.cpp), is this an issue?
  - Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, ...
    - F(n) = F(n-1) + F(n-2), why isn’t this enough? What’s needed?
    - Classic example of bad recursion, to compute F(6), the sixth Fibonacci number, we must compute F(5) and F(4). What do we do to compute F(5)? Why is this a problem?
  - Towers of Hanoi
    - N disks on one of three pegs, transfer all disks to another peg, never put a disk on a smaller one, only on larger
    - Every solution takes “forever” when N, number of disks, is large

Fibonacci: Don’t do this recursively

```c
long RecFib(int n)
// precondition: 0 <= n
// postcondition: returns the n-th Fibonacci number
{
    if (0 == n || 1 == n) {
        return 1;
    } else {
        return RecFib(n-1) + RecFib(n-2);  
    }
}
```

- How many calls of RecFib(0)?
- How many total calls?
- How many clones/calls to compute F(5)?

See recfib2.cpp for caching code
Towers of Hanoi

- The origins of the problem/puzzle may be in the far east
  - Move \( n \) disks from one peg to another in a set of three

```cpp
void Move(int from, int to, int aux, int numDisks)
// pre: numDisks on peg # from, move to peg # to
// post: disks moved from peg 'from' to peg 'to' via 'aux'
{
    if (numDisks == 1) {
        cout << "move " << from << " to " << to << endl;
    } else {
        Move (from,aux,to,numDisks - 1);
        Move (from,to,aux, 1);
        Move (aux,to,from,numDisks - 1);
    }
}
```

What's better: recursion/iteration?

- There's no single answer, many factors contribute
  - Ease of developing code assuming other factors ok
  - Efficiency (runtime or space) can matter, but don't worry about efficiency unless you know you have to
- In some examples, like Fibonacci numbers, recursive solution does extra work, we'd like to avoid the extra work
  - Iterative solution is efficient
  - The recursive inefficiency of "extra work" can be fixed if we remember intermediate solutions: static variables

- Static function variable: maintain value over all function calls
  - Local variables constructed each time function called

Fixing recursive Fibonacci: recfib2.cpp

```cpp
long RecFib(int n)
// precondition: 0 <= n <= 30
// postcondition: returns the n-th Fibonacci number
{
    static vector<int> storage(31,0);
    if (0 == n || 1 == n) return 1;
    else if (storage[n] != 0) return storage[n];
    else {
        storage[n] = RecFib(n-1) + RecFib(n-2);
        return storage[n];
    }
}
```

Thinking recursively

- Problem: find the largest element in a vector
  - Iteratively: loop, remember largest seen so far
  - Recursive: find largest in \([1..n)\), then compare to 0\textsuperscript{th} element

```cpp
double Max(const vector<double>& a)
// pre: a contains a.size() elements, 0 < a.size()
// post: return maximal element of a
{
    int k;
    double max = a[0];
    for(k=0; k < a.size(); k++) {
        if (max < a[k]) max = a[k];
    }
    return max;
}
```
### Recursive Max

```cpp
double RecMax(const tvector<double>& a, int first)
// pre: a contains a.size() elements, 0 < a.size()
// first < a.size()
// post: return maximal element a[first..size()-1]
{
    if (first == a.size()-1) { // last element, done
        return a[first];
    }
    double maxAfter = RecMax(a, first+1);
    if (maxAfter < a[first]) return a[first];
    else return maxAfter;
}
```

- What is base case (conceptually)?
- We can use RecMax to implement Max as follows
  ```cpp```
  ```
  return RecMax(a, 0);
  ```
  ```
```

### Recognizing recursion:

```cpp
void Change(tvector<int>& a, int first, int last)
// post: a is changed
{
    if (first < last)
    {
        int temp = a[first];  // swap a[first], a[last]
        a[first] = a[last];
        a[last] = temp;
        Change(a, first+1, last-1);
    }
}
```

- What is base case? (no recursive calls)
- What happens before recursive call made?
- How is recursive call closer to the base case?

### More recursion recognition

```cpp
int Value(const tvector<int>& a, int index)
// pre: ??
// post: a value is returned
{
    if (index < a.size())
    {
        return a[index] + Value(a, index+1);
    }
    return 0;
}
```

- What is base case, what value is returned?
- How is progress towards base case realized?
- How is recursive value used to return a value?
- What if a is vector of doubles, does anything change?

### One more recognition

```cpp
void Something(int n, int& rev)
// post: rev has some value based on n
{
    if (n != 0) {
        rev = (rev*10) + (n % 10);
        Something(n/10, rev);
    }
}
```

```cpp
int Number(int n)
{
    int value = 0;
    Something(n, value);
    return value;
}
```

- What is returned by `Number(13)`?
- `Number(1234)`?
  - This is a tail recursive function, last statement recursive
  - Can turn tail recursive functions into loops very easily

```cpp
```
Non-recursive version

```cpp
int Number(int n)
// post: return reverse of n, e.g., 4231 for n==1234
{
    int rev = 0;    // rev is reverse of n's digits so far
    while (n != 0)
    {
        rev = (rev * 10) + n % 10;
        n /= 10;
    }
}
```

- Why did recursive version need the helper function?
  - Where does initialization happen in recursion?
  - How is helper function related to idea above?
- Is one of these easier to understand?
- What about developing rather than recognizing?

Two-Dimensional vectors/matrices

- Many problems lead to grids, boards, matrices
  - Represent images as rectangular grid of pixels
    - What about 3-D images?
    - What's in a pixel?
  - Represent boards/boards as 2-D collection
    ```cpp
tvector<string> ttt(3,"XXX");
    ttt[0] = "X ";
    ttt[1] = " X ";
    ttt[2] = " ";
    ttt[2][2] = 'X';    // we have a winner!
```
  - Why are there two indices used with ttt?

More on grids, vectors, matrices

```cpp
tvector<double> line(30,0.0);
tvector<tvector<double> > grid(20,line);
for(int k=0; k < 20; k++)
{
    grid[k][5] = 4.0;
}
```

- How is # rows specified? Columns?
  - How do we set row with index 8 to have value 2.0?
    ```cpp
    for(int k=0; k < 30; k++)
    { grid[8][k] = 2.0;
    }
    ```

The class tmatrix and 2-D arrays

- Represents grid, similar to vector
  - Two-dimensional, specify both dimensions
  - Rather than using size(), use numrows(), numcols()
  - A vector<vector< ... > > is similar, and standard
- Can also specify grid using standard array-notation
  ```cpp
  int x[] = {1,2,3,4,5};    // like tvector of 5 elements
  int grid[20][30];        // like matrix, but fixed in size
  ```
- If the grid is global, initialized to all 0's, otherwise need ...
  ```cpp
  int x[] = {1,2,3,4,5};    // like tvector of 5 elements
  for(int k=0; k < 30; k++)
  { grid[3][k] = 0;
  } 
```
Blob Counting: Recursion at Work

- Blob counting is similar to what's called Flood Fill, the method used to fill in an outline with a color (use the paint-can in many drawing programs to fill in)
  - Possible to do this iteratively, but hard to get right
  - Simple recursive solution

- Suppose a slide is viewed under a microscope
  - Count images on the slide, or blobs in a gel, or ...
  - Erase noise and make the blobs more visible

- To write the program we'll use a class CharBitMap which represents images using characters
  - The program blobs.cpp and class Blobs are essential too

Counting blobs, the first slide

```
prompt> blobs
enter row col size 10 50
# pixels on: between 1 and 500: 200
+--------------------------------------------------+
| *  *    *    *  *  *  ***     *  ****   * *      ||    * *        ***   **   **  * * *       *  * * *||  * *** * * *** *  *    *   *    * *    *   *   **|
|   *       **  ** *         **  * *  * ***    * * |
| *   *   ***** ***  *         *       **     ** * ||* *    *        * *    * **   *     ***  *  ***  *||* *            *** * **     * *  *    * *      ** ||*  *    **  *    *   *     *  *** **    *         |
| ****    *  *   **  ****    *   ***    * *      **|
|**  *  *       *    ** **** **        *  * **    *|+--------------------------------------------------+
```

- How many blobs are there? Blobs are connected horizontally and vertically, suppose a minimum of 10 cells in a blob
  - What if blob size changes?

Identifying Larger Blobs

```
blob size (0 to exit) between 0 and 50: 10
.................................1..........................
.............................111................................
...............................1..............111.......3333.2.
...............................111..............3333322..........6
.................................2..............444...555
.................................2..............6
.................................8
.................................2
.................................7
.................................666.66
.................................8888
.................................22.7777
.................................666
.................................88
.................................8
.................................77.7777
# blobs = 3
```

- The class Blobs makes a copy of the CharBitMap and then counts blobs in the copy, by erasing noisy data (essentially)
  - In identifying blobs, too-small blobs are counted, then uncounted by erasing them

Identifying smaller blobs

```
blob size (0 to exit) between 0 and 50: 5
.................................1..........................
...........................1.1............22222222...........
...........................333.22                44444444
...........................3333.22                55555555
...........................22222222                66666666
...........................44444444                77777777
...........................66666666                88888888
...........................77777777
# blobs = 8
```

- What might be a problem if there are more than nine blobs?
  - Issues in looking at code: how do language features get in the way of understanding the code?
  - How can we track blobs, e.g., find the largest blob?
**Issues that arise in studying code**

- What does static mean, values defined in Blobs?
  - Class-wide values rather than stored once per object
  - All Blob variables would share PIXEL_OFF, unlike myBlobCount which is different in every object
  - When is static useful?

- What is the class tmatrix?
  - Two-dimensional vector, use a[0][1] instead of a[0]
  - First index is the row, second index is the column

- We’ll study these concepts in more depth, a minimal understanding is needed to work on blobs.cpp

**Recursive helper functions**

- Client programs use Blobs::FindBlobs to find blobs of a given size in a CharBitMap object
  - This is a recursive function, private data is often needed/used in recursive member function parameters
  - Use a helper function, not accessible to client code, use recursion to implement member function

- To find a blob, look at every pixel, if a pixel is part of a blob, identify the entire blob by sending out recursive clones/scouts
  - Each clone reports back the number of pixels it counts
  - Each clone “colors” the blob with an identifying mark
  - The mark is used to avoid duplicate (unending) work

**Conceptual Details of BlobFill**

- Once a blob pixel is found, four recursive clones are “sent out” looking horizontally and vertically, reporting pixel count
  - How are pixel counts processed by clone-sender?
  - What if all the clones ultimately report a blob that’s small?

- In checking horizontal/vertical neighbors what happens if there aren’t four neighbors? Is this a potential problem?
  - Who checks for valid pixel coordinates, or pixel color?
  - Two options: don’t make the call, don’t process the call

- Non-recursive member function takes care of looking for blobsign, then filling/counting/unfilling blobs
  - How is unfill/uncount managed?

**Saving blobs**

- In current version of Blobs::FindBlobs the blobs are counted
  - What changes if we want to store the blobs that are found?
  - How can clients access the found blobs?
  - What is a blob, does it have state? Behavior?
  - What happens when a new minimal blob size is specified?

- Why are the Blob class declaration, member function implementations, and main function in one file?
  - Advantages in using? blobs.h, blobs.cpp, doblobs.cpp?
  - How does Makefile or IDE take care of managing multiple file projects/programs?