Solving Problems Recursively

- **Recursion is an indispensable tool in a programmer’s toolkit**
  - Allows many complex problems to be solved simply
  - Elegance and understanding in code often leads to better programs: easier to modify, extend, verify (and sometimes more efficient!! See expotest.cpp)
  - Sometimes recursion isn’t appropriate, when it’s bad it can be very bad---every tool requires knowledge and experience in how to use it

- **The basic idea is to get help solving a problem from coworkers (clones) who work and act like you do**
  - Ask clone to solve a simpler but similar problem
  - Use clone’s result to put together your answer
- **Need both concepts: call on the clone and use the result**
Print words entered, but backwards

- **Can use a vector, store all the words and print in reverse order**
  - The vector is probably the best approach, but recursion works too

```cpp
void PrintReversed()
{
    string word;
    if (cin >> word) {    // reading succeeded?
        PrintReversed();    // print the rest reversed
        cout << word << endl;    // then print the word
    }
}

int main()
{
    PrintReversed();
}
```

- **The function PrintReversed reads a word, prints the word only after the clones finish printing in reverse order**
  - Each clone has its own version of the code, its own `word` variable
Exponentiation

- Computing $x^n$ means multiplying $n$ numbers (or does it?)
  - What’s the easiest value of $n$ to compute $x^n$?
  - If you want to multiply only once, what can you ask a clone?

```java
double Power(double x, int n)
// post: returns $x^n$
{
    if (n == 0)
    {
        return 1.0;
    }
    return x * Power(x, n-1);
}
```

- What about an iterative version?
Faster exponentiation

- How many recursive calls are made to computer $2^{1024}$?
  - How many multiplies on each call? Is this better?

```java
double Power(double x, int n)
// post: returns x^n
{
    if (n == 0) {
        return 1.0;
    }
    double semi = Power(x, n/2);
    if (n % 2 == 0) {
        return semi*semi;
    }
    return x * semi * semi;
}
```

- What about an iterative version of this function?
In the program expotest.cpp we calculate $x^n$ using $\log(n)$ multiplications (basically). We do this both iteratively and recursively using BigInt variables.

- We saw the iterative code in Chapter 5 with doubles.
- BigInt has overloaded operators so these values work like ints and doubles.
- We use the CTimer class to time the difference in using these functions (ctimer.h).

The recursive version is faster, sometimes much faster.

- Using doubles we wouldn’t notice a difference.
- Artifact of algorithm? Can we “fix the problem”?
- Natural version of both in programs, optimizing tough.
Back to Recursion

- **Recursive functions have two key attributes**
  - There is a *base case*, sometimes called the *exit case*, which does not make a recursive call
    - See print reversed, exponentiation, factorial for examples
  - All other cases make a recursive call, with some parameter or other measure that decreases or moves towards the base case
    - Ensure that sequence of calls eventually reaches the base case
    - “Measure” can be tricky, but usually it’s straightforward

- **Example: sequential search in a vector**
  - If first element is search key, done and return
  - Otherwise look in the “rest of the vector”
  - How can we recurse on “rest of vector”?
Classic examples of recursion

- **For some reason, computer science uses these examples:**
  - Factorial: we can use a loop or recursion (see `facttest.cpp`), is this an issue?
  - Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, ...
    - F(n) = F(n-1) + F(n-2), why isn’t this enough? What’s needed?
    - Classic example of bad recursion, to compute F(6), the sixth Fibonacci number, we must compute F(5) and F(4). What do we do to compute F(5)? Why is this a problem?
  - Towers of Hanoi
    - N disks on one of three pegs, transfer all disks to another peg, never put a disk on a smaller one, only on larger
    - Every solution takes “forever” when N, number of disks, is large
Fibonacci: Don’t do this recursively

```cpp
long RecFib(int n)
// precondition: 0 <= n
// postcondition: returns the n-th Fibonacci number
{
    if (0 == n || 1 == n) {
        return 1;
    }
    else {
        return RecFib(n-1) + RecFib(n-2);
    }
}
```

- How many clones/calls to compute F(5)?

```
How many calls of F(1)?
How many total calls?

See recfib2.cpp for caching code
```
Towers of Hanoi

- The origins of the problem/puzzle may be in the far east
  - Move n disks from one peg to another in a set of three

```cpp
void Move(int from, int to, int aux,
           int numDisks)
// pre: numDisks on peg # from,
//      move to peg # to
// post: disks moved from peg 'from'
//       to peg 'to' via 'aux'
{
    if (numDisks == 1) {
        cout << "move " << from << " to " << to << endl;
    }
    else {
        Move (from,aux,to, numDisks - 1);
        Move (from,to,aux, 1);
        Move (aux,to,from, numDisks - 1);
    }
}
```
What’s better: recursion/iteration?

- There’s no single answer, many factors contribute
  - Ease of developing code assuming other factors ok
  - Efficiency (runtime or space) can matter, but don’t worry about efficiency unless you know you have to
- In some examples, like Fibonacci numbers, recursive solution does extra work, we’d like to avoid the extra work
  - Iterative solution is efficient
  - The recursive inefficiency of “extra work” can be fixed if we remember intermediate solutions: static variables

- Static function variable: maintain value over all function calls
  - Local variables constructed each time function called
Fixing recursive Fibonacci: recfib2.cpp

```cpp
long RecFib(int n) {
    // precondition: 0 <= n <= 30
    // postcondition: returns the n-th Fibonacci number
    static tvector<int> storage(31,0);

    if (0 == n || 1 == n)     return 1;
    else if (storage[n] != 0) return storage[n];
    else {
        storage[n] = RecFib(n-1) + RecFib(n-2);
        return storage[n];
    }
}
```

- **What does storage do? Why initialize to all zeros?**
  - Static variables initialized first time function called
  - Maintain values over calls, not reset or re-initialized
Thinking recursively

- **Problem:** find the largest element in a vector
  - **Iteratively:** loop, remember largest seen so far
  - **Recursive:** find largest in [1..n), then compare to 0th element

```cpp
double Max(const tvectordouble>& a)
// pre: a contains a.size() elements, 0 < a.size()
// post: return maximal element of a
{
    int k;
    double max = a[0];
    for(k=0; k < a.size(); k++) {
        if (max < a[k]) max = a[k];
    }
    return max;
}
```

- In a recursive version what is base case, what is measure of problem size that decreases (towards base case)?
Recursive Max

double RecMax(const tvector<double>& a, int first)
// pre: a contains a.size() elements, 0 < a.size()
//      first < a.size()
// post: return maximal element a[first..size()-1]
{
    if (first == a.size()-1){ // last element, done
        return a[first];
    }
    double maxAfter = RecMax(a,first+1);
    if (maxAfter < a[first]) return a[first];
    else return maxAfter;
}

● What is base case (conceptually)?
● We can use RecMax to implement Max as follows

return RecMax(a,0);
Recognizing recursion:

```c
void Change(tvector<int>& a, int first, int last)
{ // post: a is changed
    if (first < last)
    {
        int temp = a[first]; // swap a[first], a[last]
        a[first] = a[last];
        a[last] = temp;
        Change(a, first+1, last-1);
    }
}
// original call (why?): Change(a, 0, a.size()-1);
```

- What is base case? (no recursive calls)
- What happens before recursive call made?
- How is recursive call closer to the base case?
More recursion recognition

```cpp
int Value(const tvector<int>& a, int index)
// pre: ??
// post: a value is returned
{
    if (index < a.size())
    {
        return a[index] + Value(a,index+1);
    }
    return 0;
}
// original call: cout << Value(a,0) << endl;
```

- What is base case, what value is returned?
- How is progress towards base case realized?
- How is recursive value used to return a value?
- What if a is vector of doubles, does anything change?
One more recognition

```c
void Something(int n, int& rev)
    // post: rev has some value based on n
    {
        if (n != 0) {
            rev = (rev*10) + (n % 10);
            Something(n/10, rev);
        }
    }

int Number(int n)
    {
        int value = 0;
        Something(n,value);
        return value;
    }
```

- **What is returned by** `Number(13)` ? `Number(1234)` ?
  - This is a tail recursive function, last statement recursive
  - Can turn tail recursive functions into loops very easily
Non-recursive version

```cpp
int Number(int n) {
    int rev = 0;  // rev is reverse of n's digits so far
    while (n != 0) {
        rev = (rev * 10) + n % 10;
        n /= 10;
    }
}
```

- **Why did recursive version need the helper function?**
- Where does initialization happen in recursion?
- How is helper function related to idea above?

- **Is one of these easier to understand?**
- **What about developing rather than recognizing?**
Two-Dimensional vectors/matrices

- Many problems lead to grids, boards, matrices
  - Represent images as rectangular grid of pixels
    - What about 3-D images?
  - What's in a pixel?

- Represent boards/grids as 2-D collection

```cpp
tvector<string> ttt(3,"XXX");
ttt[0] = "X ";
ttt[1] = " X ";
ttt[2] = "   ";
ttt[2][2] = 'X';    // we have a winner!
```

- Why are there two indices used with `ttt`?
More on grids, vectors, matrices

tvector<double> line(30,0.0);
tvector<tvector<double>> grid(20,line);
for(int k=0; k < 20; k++){
  grid[k][5] = 4.0;
}

How is # rows specified? Columns?

- How do we set row with index 8 to have value 2.0?

  for(int k=0; k < 30; k++){
    grid[8][k] = 2.0;
  }
The class tmatrix and 2-D arrays

- Represents grid, similar to tvector
  - Two-dimensional, specify both dimensions
  - Rather than using size(), use numrows(), numcols()
  - A `vector<vector<...>>` is similar, and standard

- Can also specify grid using standard array-notation

```c
int x[] = {1,2,3,4,5}; // like tvector of 5 elements
int grid[20][30];      // like matrix, but fixed in size
```

- If the grid is global, initialized to all 0's, otherwise need ...

```c
for(int j=0; j < 20; j++)
    for(int k=0; k < 30; k++)
        grid[j][k] = 0;
```
Blob Counting: Recursion at Work

- Blob counting is similar to what’s called Flood Fill, the method used to fill in an outline with a color (use the paint-can in many drawing programs to fill in)
  - Possible to do this iteratively, but hard to get right
  - Simple recursive solution

- Suppose a slide is viewed under a microscope
  - Count images on the slide, or blobs in a gel, or ...
  - Erase noise and make the blobs more visible

- To write the program we’ll use a class CharBitMap which represents images using characters
  - The program blobs.cpp and class Blobs are essential too
Counting blobs, the first slide

prompt> blobs
enter row col size 10 50
# pixels on:  between 1 and 500: 200

- How many blobs are there? Blobs are connected horizontally and vertically, suppose a minimum of 10 cells in a blob
  - What if blob size changes?
Identifying Larger Blobs

blob size (0 to exit) between 0 and 50: 10
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Identifying smaller blobs

blob size (0 to exit) between 0 and 50: 5

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......................8.............2..7.............666.66

......................8888....................22..7777......666

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# blobs = 8

- What might be a problem if there are more than nine blobs?
  - Issues in looking at code: how do language features get in the way of understanding the code?
  - How can we track blobs, e.g., find the largest blob?
Issues that arise in studying code

- **What does static mean, values defined in Blobs?**
  - Class-wide values rather than stored once per object
  - All Blob variables would share PIXEL_OFF, unlike myBlobCount which is different in every object
  - When is static useful?

- **What is the class tmatrix?**
  - Two-dimensional vector, use \[ a[0][1] \] instead of \[ a[0] \]
  - First index is the row, second index is the column

- **We’ll study these concepts in more depth, a minimal understanding is needed to work on blobs.cpp**
Recursive helper functions

- **Client programs use** `Blobs::FindBlobs` **to find blobs of a given size in a CharBitMap object**
  - This is a recursive function, private data is often needed/used in recursive member function parameters
  - Use a helper function, not accessible to client code, use recursion to implement member function

- **To find a blob, look at every pixel, if a pixel is part of a blob, identify the entire blob by sending out recursive clones/scouts**
  - Each clone reports back the number of pixels it counts
  - Each clone “colors” the blob with an identifying mark
  - The mark is used to avoid duplicate (unending) work
Conceptual Details of BlobFill

- Once a blob pixel is found, four recursive clones are “sent out” looking horizontally and vertically, reporting pixel count
  - How are pixel counts processed by clone-sender?
  - What if all the clones ultimately report a blob that’s small?

- In checking horizontal/vertical neighbors what happens if there aren’t four neighbors? Is this a potential problem?
  - Who checks for valid pixel coordinates, or pixel color?
  - Two options: don’t make the call, don’t process the call

- Non-recursive member function takes care of looking for blobsign, then filling/counting/unfilling blobs
  - How is unfill/uncount managed?
Saving blobs

- In current version of `Blobs::FindBlobs` the blobs are counted
  - What changes if we want to store the blobs that are found?
  - How can clients access the found blobs?
  - What is a blob, does it have state? Behavior?
  - What happens when a new minimal blob size is specified?

- Why are the Blob class declaration, member function implementations, and main function in one file?
  - Advantages in using `blobs.h`, `blobs.cpp`, `doblobs.cpp`?
  - How does Makefile or IDE take care of managing multiple file projects/programs?