Due: Thursday, Feb. 11
25 points

On homework, you may discuss with other students in the course about how to solve a problem, but the write-up should be your own. You must include the names of any students you consulted with. Give credit where credit is due.

1. (3 pts) Consider the following NFA. Give an equivalent regular expression.

2. (3 pts) Give a regular grammar for the following DFA.

3. (9 pts) Construct a regular grammar for the following languages.

   (a) \( L = \{a^n b^m c^p \mid n \text{ is even, } m > 0 \text{ and } p \text{ is odd} \} \)

   (b) \( L = \{w \in \Sigma^* \mid w \text{ has an even number of } a \text{'s and an odd number of } b \text{'s} \} \)

   Note that abbaaba \( \in L \).

   (c) \( L = \{w \in \Sigma^* \mid (n_a(w) - n_b(w)) \mod 3 = 1 \} \), where \( n_a(w) \) is the number of \( a \)'s in \( w \).

4. (5 pts) Define \( \text{Suf}(L) = \{w \in \Sigma^* \mid x = yw \text{ for some } x \in L, y \in \Sigma^* \} \) (the set of suffixes of \( L \))
Prove that $\text{Suf}(L)$ is closed under the regular languages. (Hint: Given a DFA for $L$, construct a DFA for $\text{Suf}(L)$.)

5. (5 pts) Consider the following property, $\text{doublefirst}_a$.

$\Sigma = \{a, b, c, d\}$

$\text{doublefirst}_a(L) = \{waav|wav \in L, v \in \Sigma^*, \text{ and } w \in \Sigma - \{a\}^*\}$

The property $\text{doublefirst}_a$ applied to a language $L$ replaces the first occurrence of $a$ in each string by $aa$. For example, if the string $babb$ is in $L$, then $baabb$ is in $\text{doublefirst}_a(L)$.

**Prove** that the regular languages are closed under the $\text{doublefirst}_a$ property.