On homework, you may discuss with other students in the course about how to solve a problem, but the write-up should be your own. You must include the names of any students you consulted with. Give credit where credit is due.

1. (12 pts) Find context-free grammars for the following.

   - L = \{a^n b^m \mid n \geq m + 4, n > 0, m > 0\}
   - L = \{a^n b^m \mid 2m \leq n \leq 3m, n > 0, m > 0\}
   - L = \{a^n b^m c^m \mid p > n + m, n \geq 0, n \text{ is even, } m \geq 0\}

2. (2 pts) Show that the following grammar is ambiguous.

   \[
   S \rightarrow AB \mid aaB \\
   A \rightarrow a \mid Aa \\
   B \rightarrow b
   \]

3. (2 pts) Construct an unambiguous grammar equivalent to the grammar in problem 2.

4. (3 pts) Eliminate useless productions from the following grammar. Use the algorithm from the lecture notes.

   \[
   S \rightarrow a \mid aA \mid B \mid C \\
   A \rightarrow aB \mid \epsilon \\
   B \rightarrow Aa \mid C \\
   C \rightarrow cCD \\
   D \rightarrow ddd
   \]

   In particular, give the set \(V_1\), the dependency graph and the new grammar.

5. (3 pts) Eliminate all \(\epsilon\)-productions from the following grammar. Use the algorithm from the lecture notes.

   \[
   S \rightarrow AaB \mid aaB \\
   A \rightarrow \epsilon \\
   B \rightarrow bbA \mid \epsilon
   \]

   In particular, give the set \(V_N\) and the new grammar.
6. (3 pts) Remove the unit productions from the following grammar. Use the algorithm from the lecture notes.

\[
\begin{align*}
S & \rightarrow a \mid aA \mid B \mid C \\
A & \rightarrow aB \mid \varepsilon \\
B & \rightarrow Aa \mid C \\
C & \rightarrow cCD \\
D & \rightarrow ddd
\end{align*}
\]

In particular, show the dependency graph, and the new grammar.

7. (3 pts) Convert the following grammar into Chomsky Normal Form.

\[
\begin{align*}
S & \rightarrow abAB \\
A & \rightarrow bAB \mid \varepsilon \\
B & \rightarrow BAa \mid A \mid \varepsilon
\end{align*}
\]