Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines.

1. Given Turing Machines $M_1$ and $M_2$
   
   Notation for
   - Run $M_1$
   - Run $M_2$

   $M_1 \rightarrow M_2$

2. Given Turing Machines $M_1$ and $M_2$
   
   Notation for
   - Run $M_1$
   - If $x$ is current symbol
     - then Run $M_2$
3. Given Turing Machines M1, M2, and M3

Notation for

- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

More Notation for Simplifying Turing Machines

Suppose $\Sigma = \{a, b, c, B\}$

- z is any symbol in $\Sigma$
- x is a specific symbol from $\Sigma$

1. s - start
2. R - move right
3. L - move left
4. x - write x (and don’t move)
5. $R_a$ - move right until you see an a
6. $L_a$ - move left until you see an a
7. $R_{\neg a}$ - move right until you see anything that is not an a
8. $L_{\neg a}$ - move left until you see anything that is not an a
9. h - halt in a final state
10. $a \rightarrow \frac{b}{w}$

If the current symbol is a or b, let w represent the current symbol.
**Example**

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

- input: $bab$, output: $babb$
- input: $ba$, output: $ba$

![Diagram](chart.png)

What is the running time?

**Example**

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$, $|w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

- input: $abbabb$, output: $abbaabbb$

The tape head should finish pointing at the leftmost symbol of $w$.

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**Turing’s Thesis** Any computation that can be carried out by a mechanical means can be performed by a TM.

**Definition:** An *algorithm* for a function $f: \mathbb{D} \rightarrow \mathbb{R}$ is a TM $M$, which given input $d \in \mathbb{D}$, halts with answer $f(d) \in \mathbb{R}$. 
**Example:** \( f(x + y) = x + y, x \) and \( y \) unary numbers.

\[
\begin{align*}
\text{start with:} & \quad 111+1111 \\
& \quad \uparrow \\
\text{end with:} & \quad 1111111 \\
& \quad \uparrow
\end{align*}
\]

**Example:** Copy a String, \( f(w) = w0w, w \in \Sigma^*, \Sigma = \{a, b, c\} \)

Denoted by \( C \)

\[
\begin{align*}
\text{start with:} & \quad abac \\
& \quad \uparrow \\
\text{end with:} & \quad abac0abac \\
& \quad \uparrow
\end{align*}
\]

**Algorithm:**

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol

![Diagram](image)
**Example:** Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

```
start with:   aaBbabca
  ↑
end with:    aaBBbaca
  ↑
```

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position

```
s  R  \(\text{a,b,c,B}\)  \(\downarrow\)  v  0
  \(\downarrow\)
  L  B  L  \(\text{a,b,c}\)  \(\downarrow\)  \(\rightarrow\)  w  B  R  w  L
  \(\downarrow\)
  B
  R  v  L  h
```

**Example:** Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

```
start with:   babcaBba
  ↑
end with:    bacaBBba
  ↑
```

(similar to $S_R$)
Example: Add unary numbers

This time use shift.

Example: Multiply two unary numbers, \( f(x*y) = x*y \), \( x \) and \( y \) unary numbers. Assume \( x,y > 0 \).

<table>
<thead>
<tr>
<th>Start with:</th>
<th>1111*11</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>↑</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>End with:</th>
<th>11111111</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>↑</td>
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