**L-system**

L-systems are grammatical systems introduced by Lyndenmayer to describe biological developments such as the growth of plants and cellular organisms.

An L-system is composed of three parts \((\Sigma, h, w)\)

- \(\Sigma\) finite alphabet set of symbols
- \(h\) rewriting rules each symbol is replaced by string of symbols
- \(w\) axiom starting point

\(h\) is finite substitutions, \(h: \Sigma \rightarrow \Sigma^*\).

\(h(w)\) is computed by replacing every symbol in \(w\) that has a rewrite rule by that rule.

A language \(L\) of an L-system is the word sequence generated by

- \(h^0(w) = w\)
- \(h^1(w) = h(w)\)
- \(h^2(w) = h(h(w))\)
- ...

\(L = \{h^i(w) \mid i \geq 0\}\)

**NOTE:** If \(h(a) = bb\) we will write this as a rule

\[ a \Rightarrow bb \]

meaning the symbol \(a\) can be replaced by the symbols \(bb\).
Example:

Σ  alphabet: \{a, b\} 

h  rules:  
    a => a 
    b => ab 

w  axiom:  ab 

Notes:

- => means “is replaced by”
- left hand side of rule must be a single character
- there is at most one rule for each character

What is the language L of strings represented by this L-system? that is, starting with the axiom, what are all the strings that can be generated from the rules?

L =

Example:

Σ  alphabet: \{a\} 

h  rules:  
    a => aa 

w  axiom:  a 

L =

**Drawing a picture of an L-system**

Defining an L-system: (3 parts in this order)

- Axiom definition: This must be the first line of the file
- Production rules: Defines the replacement rules.
  There must be spaces between the symbols on the right hand side of rules.
- Geometric rules: Defines colors, widths, etc.

Symbols for drawing and moving:

- g: draw a line one step in the current direction
- f: move forward one step in the current direction
Example:

This example is in the file: samp1

\[
\begin{align*}
\text{SET axiom X} & \quad \text{axiom definition} \\
X & \Rightarrow g f g & \text{production rule} \\
\text{SET d 15} & \quad \text{geometric rules} \\
\text{SET iniwidth 5} & \quad \text{length of line drawn is 15 units} \\
\text{SET color black} & \quad \text{width of initial line is 5 units}
\end{align*}
\]

Example:

This example is in the file: samp2

\[
\begin{align*}
\text{SET axiom X} & \quad \text{start symbol is X} \\
X & \Rightarrow g f g X & \text{only change from previous program, repetition} \\
\text{SET d 15} & \quad \text{length of line} \\
\text{SET iniwidth 5} & \quad \text{width of initial line} \\
\text{SET color black}
\end{align*}
\]
Symbols for changing direction

- +: change direction to the right in a determined angle
- -: change direction to the left in a determined angle
- &: change direction pitch down in a determined angle
- A: change direction pitch up in a determined angle
- \times: change direction roll left in a determined angle
- /\: change direction roll right in a determined angle
- \%: change direction 180 degrees

Example:

This example is in the file: samp3

```
SET axiom X start symbol is X
X ==> g g g X + Y  rule with 2 replacements
Y ==> g

SET d 15  length of line
SET iniwidth 5 width of initial line
SET angle 15 angle for change of direction
SET color blue  initial color
```

L =

Example:

We will make just a slight change in the L-system. This example is in the file: samp4

```
```
SET axiom X

\[
X \Rightarrow g \text{ change blue } g \text{ g change yellow } + Y X
\]

\[
Y \Rightarrow g
\]

only difference with file samp3

SET d 15
SET initwidth 5
SET angle 15
SET color black

L =

Stacking operations

- [: save in stack status of turtle which is current direction position and width of line
- [: recover from stack status of turtle

To make a branch, the turtle must draw one part of the branch and then come back to the fork position
and draw the other part of the branch. Part of a string can be saved for processing by putting it within
brackets [ ].

Example, consider the rules

SET axiom X

\[
X \Rightarrow g [ \sim + Y g ] g
\]

\[
Y \Rightarrow + Y
\]

within [ ]'s is a branch

the \sim \text{ means decrement the width of the line}

SET d 18
SET initwidth 4
SET incwidth 1
SET angle 30
SET color black

First string in \( L \) is \( g[\sim + Y g]g \). To draw this first draw the first line for the first \( g \). At this point, save the
\( [\sim + Y g] \) along with the current direction and the current width of the line. Continue drawing at the first
symbol past the \( ] \). Draw a line. Now that the end of the string has been reached, come back to the point in
the drawing where the branch occurred and draw the string within the \( [ ] \)'s.

What is \( L \)? \( L = \)

Suppose we change the \( X \) rule above by adding \( X \) onto the end:

\[
X \Rightarrow g [ \sim + Y g ] g X
\]
Now the L-system looks like:

```
Example of drawing plants via L-systems
Now we will examine some examples of growing plants.
Example:
This is in file: plant1

SET axiom X
X ==> g [ ~ + g Y ] g X
Y ==> g g [ ~ + g leaf ]
leaf ==> [ color Green { + f - f f - f + % + f f f } ]

SET d 18
SET initwidth 4
SET incwidth 1
SET angle 18
SET color black

Differences from previous examples:

- The color field allows you to specify a color for part of the drawing.
- The parenthesis { } are used to define a region that is to be filled in.

Example:
This is in file: plant2

SET axiom X

\[ X \rightarrow g [ \sim + g Y ] [ \sim + g \ Y \sim ] \sim g X \]
\[ Y \rightarrow X \sim g [ \sim + g \text{ leaf} ] \]
leaf \( \rightarrow [ \color{Green} \{ + f \ - f f - f + f f - f \} ] \)

SET d 18
SET iniwidth 4
SET incwidth 1
SET angle 18
SET color black

Fractals

References:

- The Algorithmic Beauty of Plants, by P. Prusinkiewicz and A. Lindenmayer
- Automata, Languages, Development, by A. Lindenmayer and G. Rozenberg