Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:

Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape
• actions
  – write symbol
  – read symbol
  – move left (L) or right (R)

• computation
  – initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  – processing computation
    * move tape head left or right
    * read from and write to tape
  – computation halts
    * final state

Formal Definition of TM

A TM M is defined by $M = (K, \Sigma, \delta, q_0, B, F)$ where

• $K$ is finite set of states
• $\Sigma$ is input alphabet
• , is tape alphabet
• $B \in \Sigma$ is blank
• $q_0$ is start state
• $F$ is set of final states
• $\delta$ is transition function
  $\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an ‘a’, then move into state $p$, write a ‘b’ on the tape and move to the right”.

TM as Language recognizer

Definition: Configuration is denoted by $\vdash$. If $\delta(q,a) = (p,b,R)$ then a move is denoted

\[
abaaqbba \vdash ababpbba
\]

Definition: Let $M$ be a TM, $M = (K, \Sigma, \delta, q_0, B, F)$. $L(M) = \{ w \in \Sigma^* | q_0 w \vdash^* x_1 q_f x_2$ for some $q_f \in F, x_1, x_2 \in \Sigma^* \}$
**TM as language acceptor**

M is a TM, w is in $\Sigma^*$,

- if $w \in L(M)$ then M halts in final state
- if $w \notin L(M)$ then either
  - M halts in non-final state
  - M doesn’t halt

**Example:**

$L = \{a^n b^n c^n | n \geq 1\}$

Is the following TM correct?

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**TM as a transducer**

TM can implement a function: $f(w) = w'$

- start with: $w$
  \[\uparrow\]
  
- end with: $w'$
  \[\uparrow\]

**Definition:** A function with domain D is *Turing-computable* or *computable* if there exists TM $M = (Q, \Sigma, \delta, q_0, B, F)$ such that

$$q_0w \xrightarrow{*} q_j f(w)$$

$q_j \in F$, for all $w \in D$. 

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**Example:**

\( f(x) = 2x \)

\( x \) is a unary number

start with: 111

\[ \uparrow \]

end with: 111111

\[ \uparrow \]

Is the following TM correct?

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**Example:**

\( L = \{ww \mid w \in \Sigma^+\}, \Sigma = \{a, b\} \)