Why Computer Science is Uglier and Prettier than Mathematics

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What is Computer Science?
What is it that distinguishes it from the separate subjects with which it is related? What is the linking thread which gathers these disparate branches into a single discipline? My answer to these questions is simple — it is the art of programming a computer. It is the art of designing efficient and elegant methods of getting a computer to solve problems, theoretical or practical, small or large, simple or complex.

C.A.R (Tony) Hoare

What can be programmed?
- What class of problems can be solved?
  - G4, 500Mhz Pentium III, Cray, pencil?
  - Alan Turing proved somethings, hypothesized others
    - Halting problem, Church-Turing thesis
- What class of problems can be solved efficiently?
  - Problems with no practical solution
    - what does practical mean?
  - Problems for which we can’t find a practical solution
    - solving one solves them all

Schedule students, minimal conflicts
- Given student requests, available teachers
  - write a program that schedules classes
  - Minimize conflicts
- Add a GUI too
  - Web interface
  - ...
  - ...

I can’t write this program because I’m too dumb

One better scenario

Another possible scenario

I can’t write this program but neither can all those famous people
The halting problem: writing DoesHalt

```cpp
def DoesHalt(const string& progname, const string& s) {
    // post: returns true if progname halts given s as input, false otherwise
    int main() {
        string f = PromptString("enter filename ");
        string s = PromptString("input for "+filename);
        if (DoesHalt(f,s)) cout << "does halt" << endl;
        else               cout << "does not halt" << endl;
    } // main

    // A compiler is a program that reads other programs as input
    // Can a word counting program count its own words?
    // DoesHalt function might simulate, analyze, ...
    // One program/function that works for any program/input
}
```

Consider the program `confuse.cpp`

```cpp
#include "halt.h"
int main() {
    string f = PromptString("enter filename ");
    if (DoesHalt(f,f)) {
        while (true) {
            // do nothing forever
        }
    }
    return 0;
}
```

We want to show writing `DoesHalt` is impossible
- Proof by contradiction:
- Assume possible, show impossible situation results

Not impossible, but impractical
- Towers of Hanoi
  - How long to move n disks?
- What combination of switches turns the light on?
  - Try all combinations, how many are there?
  - Is there a better way?

Travelling Salesperson
- Visit every city exactly once
- Minimize cost of travel or distance
- Is there a tour for under $2,000? less than 6,000 miles?
- Is close good enough?

Try all paths, from every starting point—how long does this take?
```
  a, b, c, d, e, f, g
  b, a, c, d, e, f, g ...
```

Complexity Classifications
- This route hits all cities for less than $2,000 — verify properties of route efficiently.
- Hard to find optimal solution
- Pack trucks with barrels, use minimal # trucks
- Ideas?

Problems are the “same hardness”: solve one efficiently, solve them all

Are hard problems easy?
- P = easy problems, NP = “hard” problems
- P means solvable in polynomial time
  - Difference between N, N^2, N^10?
- NP means non-deterministic, polynomial time
  - guess a solution and verify it efficiently
- Question: P = NP?
  - if yes, a whole class of difficult problems can be solved efficiently—one problem is reducible to another
  - if no, none of the hard problems can be solved efficiently
  - showing the first problem was in NP was an exercise in intellectual bootstrapping (1971)
Theory and Practice

- Number theory: pure mathematics
  - How many prime numbers are there?
  - How do we factor?
  - How do we determine primeness?
- Computer Science
  - Primality is “easy”
  - Factoring is “hard”
  - Encryption is possible

Shafi Goldwasser

- RCS professor of computer science at MIT
  - Co-inventor of zero-knowledge proof protocols
  - Consider card readers for dorms
  - Access without tracking

Why is programming fun?

What delights may its practitioner expect as a reward?
First is the sheer joy of making things

Second is the pleasure of making things that are useful
Third is the fascination of fashioning complex puzzle-like objects of interlocking moving parts

Fourth is the joy of always learning
Finally, there is the delight of working in such a tractable medium. The programmer, like the poet, works only slightly removed from pure thought-stuff.

Fred Brooks

What is computer science?

- What is a computation?
  - Can formulate this precisely using mathematics
  - Can say “anything a computer can compute”
  - Study both theoretical and empirical formulations, build machines as well as theoretical models
- How do we build machines and the software that runs them?
  - Hardware: gates, circuits, chips, cache, memory, disk,...
  - Software: operating systems, applications, programs
- Art, Science, Engineering
  - How do we get better at programming and dealing with abstractions
  - What is hard about programming?

Simple, Elegant, Deep

\[(a+b)^2 = c^2 + 4\left(\frac{1}{2}ab\right)\]

\[a^2 + 2ab + b^2 = c^2 + 2ab\]

\[a^2 + b^2 = c^2\]

The selection problem

- In a list of N ordered items, find the kth smallest
  - Highest salary, median, 90%, ...
  - Solve the problem, then solve it efficiently
- Suppose we can re-arrange the items

Where is kth smallest?
- It’s x
- It’s before x
- It’s after x
The Selection Problem

- In a list of N ordered items, find the k\textsuperscript{th} smallest
  - Highest salary, median, 90%, ...
  - Solve the problem, then solve it efficiently

\begin{verbatim}
double Select(vector<double> & a, int left, int right, int k)
  // pre: left <= right
  // post: return k\textsuperscript{th} smallest in a[left..right]
  {
    if (left == right) return a[left];
    int p = Partition(a, left, right); // re-arrange a
    int size = p – left + 1;     // left sublist
    if (k <= size) return Select(a, left, p, k);
    else       return Select(a, p+1, right, k-size);
  }
\end{verbatim}

Partition: the picture and invariant

\begin{verbatim}
Partition(vector<int> & a, int left, int right)
  // pre: left <= right, and legal indices for a
  // post: return index and re-arrange elements in a
  //       so a[left..index] <= a[index+1..right]
  {
    int p=left;
    int k;
    for(k=left+1; k <= right; k++)
    {
      if (a[k] <= a[left])
      {
        p++;
        Swap(a[k], a[p]);
      }
    }
    Swap(a[left],a[p]);
    return p;
  }
\end{verbatim}

QuickSort: the code

\begin{verbatim}
void QuickSort(vector<int> & a, int left, int right)
  // pre: left <= right, and legal indices for a
  // post: a[left] <= … <= a[right]
  {
    if (left <= right)
    {
      int p = Partition(a,left,right);
      QuickSort(a,left, p-1);
      QuickSort(a, p+1, right);
    }
  }
\end{verbatim}

Practice with invariants

- Remove zeros from an array, leave order of non-zeros unchanged (AP exam, 1987)

- Sketch/write a solution
  - Make it run, make it right, make it fast

C.A.R. (Tony) Hoare (b. 1934)

- Won Turing award in 1980
- Invented quicksort, but didn’t see how simple it was to program recursively
- Developed mechanism and theory for concurrent processing
- In Turing Award speech used “Emporer’s New Clothes” as metaphor for current fads in programming

"Beginning students don’t know how to do top-down design because they don’t know which end is up"
Shoulders of Giants

Newton: “If I have seen farther it is because I have stood on the shoulders of giants.”

Robert Burton: “a dwarf standing on the shoulders of a giant may see farther than the giant himself.”

Richard Stallman (born 1953)

- Described by some as “world's best programmer”
  - Wrote/developed GNU software tools, particularly g++
  - Believes all software should be free, but like “free speech”, not “free beer”
  - Won MacArthur award for his efforts and contributions
  - League for Programming Freedom
- Gnu/Linux is a free operating system and computing environment
  - Heavy industry/web use
  - Wintel killer??

- Local tie-in: Red Hat Linux,
  - headquartered in Durham, NC
  - IPO in 1999 at $14
  - One month later at $110+
  - Markets “free” product