Towers of Hanoi

- Move disks from "from" peg to "to" peg
- What is the recurrence relation in terms of numDisks?

```cpp
void Move(int from, int to, int aux, int numDisks)
// pre: numDisks on peg from,
// post: numDisks moved to peg to
{
    if (numDisks == 1)
    {
        cout << from << " to " << to << endl;
    }
    else
    {
        Move(from, aux, to, numDisks-1);
        Move(from, to, aux, 1);
        Move(aux, to, from, numDisks-1);
    }
}
```
Backtracking, Search, Heuristics

- Many problems require an approach similar to solving a maze
  - Certain mazes can be solved using the “right-hand” rule
  - Other mazes, e.g., with islands, require another approach
  - If you have “markers”, leave them at intersections, don’t explore the same place twice

- What happens if you try to search the web, using links on pages to explore other links, using those links to …
  - How many web pages are there?
  - What rules do webcrawlers/webspiders follow?
    - Who enforces the rules?
- Keep track of where you’ve been don’t go there again
  - Any problems with this approach?
Classic problem: N queens

- Can queens be placed on a chess board so that no queens attack each other?
  - Easily place two queens
  - What about 8 queens?
- Make the board NxN, this is the N queens problem
  - Place one queen/column
  - # different tries/column?
- N Queens Demo Applet!
- Backtracking
  - Use “current” row in a col
  - If ok, try next col
  - If fail, back-up, next row
Backtracking idea with N queens

- Try to place a queen in each column in turn
  - Try first row in column $C$, if ok, move onto next column
  - If solved, great, otherwise try next row in column $C$, place queen, move onto the next column
    - Must unplace the placed queen to keep going

- What happens when we start in a column, where to start?
  - If we fail, move back to previous column (which remembers where it is/failed)
  - When starting in a column anew, start at beginning
    - When backing up, try next location, not beginning

- Backtracking in general, record an attempt go forward
  - If going forward fails, undo the record and backup
Basic ideas in backtracking search

- We need to be able to enumerate all possible choices/moves
  - We try these choices in order, committing to a choice
  - If the choice doesn’t pan out we must undo the choice
    - This is the backtracking step, choices must be undoable

- Process is inherently recursive, so we need to know when the search finishes
  - When all columns tried in N queens
  - When we have found the exit in a maze
  - When every possible moved tried in Tic-tac-toe or chess?
    - Is there a difference between these games?

- Summary: enumerate choices, try a choice, undo a choice, this is brute force search: try everything
bool Queens::SolveAtCol(int col)
   // pre: queens placed at columns 0,1,...,col-1
   // post: returns true if queen can be placed in column col
   //       and N queen problem solved (N is square board size)
   {
      int k; int rows = myBoard.numrows();
      if (col == rows) return true;

      for(k=0; k < rows; k++)
      {
         if (NoQueensAttackingAt(k,col))
            {
               myBoard[k][col] = true; // place a queen
               if (SolveAtCol(col+1))
               {
                  return true;
               }
               myBoard[k][col] = false; // unplace the queen
            }
      }
      return false;
   }
Computer v. Human in Games

- Computers can explore a large search space of moves quickly
  - How many moves possible in chess, for example?

- Computers cannot explore every move (why) so must use heuristics
  - Rules of thumb about position, strategy, board evaluation
  - Try a move, undo it and try another, track the best move

- What do humans do well in these games? What about computers?
  - What about at Duke?
Backtracking, minimax, game search

- We’ll use tic-tac-toe to illustrate the idea, but it’s a silly game to show the power of the method
  - What games might be better? Problems?

- Minimax idea: two players, one maximizes score, the other minimizes score, search complete/partial game tree for best possible move
  - In tic-tac-toe we can search until the end-of-the game, but this isn’t possible in general, why not?
  - Use static board evaluation functions instead of searching all the way until the game ends

- Minimax leads to alpha-beta search, then to other rules and heuristics
Minimax for tic-tac-toe

- Players alternate, one might be computer, one human (or two computer players)

- Simple rules: win scores +10, loss scores –10, tie is zero
  - X maximizes, O minimizes

- Assume opponent plays smart
  - What happens otherwise?

- As game tree is explored is there redundant search?
  - What can we do about this?
The words above represent a simple substitution cipher
- Each letter mapped to one other letter, no inconsistencies
- Often used in cryptogram puzzles (newspaper, online, ...)
- How can we write a computer program to solve this?

Ideas for solving the problem? Benchmark/ballpark idea to accept (or not)

Problems on the horizon?
One possible solution in docrypto.cpp

- **Study this for an example of backtracking**
  - Similar to N queens: make move, recurse, undo as needed
  - What’s a move in this problem?

- **Illustrates a few C++ and OO concepts**
  - Static variables and functions: belong to class not object
  - Also called “class variables”, don’t need object to access
  - Must be careful when initializing static variables because order of initialization can be important

- **See WordSource object shared by all CryptoMap objects, how and when is the WordSource initialized?**
Heuristics

- A heuristic is a rule of thumb, doesn’t always work, isn’t guaranteed to work, but useful in many/most cases
  - Search problems that are “big” often can be approximated or solved with the right heuristics

- What heuristic is good for cryptograms?
  - Solve small words first
  - Solve large words first
  - Do something else?

- What other optimizations/improvements can we make?
  - See program, cryptomap.cpp and docrypto.cpp