CPS 230 Homework-5

Write the solution to each problem on a single page of a separate sheet of paper.
The deadline for handing in solutions is November 18th.

1. Planar Graph Problem: (20 points)
   Let $G$ be a simple connected bipartite planar graph with $v$ vertices and $e$ edges. Prove that if $v \geq 3$, then $e \leq 2v - 4$.

2. Planar Graph Problems 2: (20 = 12 + 8 points)
   (a) Prove that every simple planar graph with $n$ vertices ($n \geq 4$) has at least four vertices with degree less than 6.
   (b) Construct an 8-vertex simple planar graph that has exactly four vertices with degree less than 6.

3. Plane-Sweep Problem: (20 points)
   A disk consists of a circle plus its interior and is represented by its center point and radius. Two disks intersect if they have a point in common. Give an $O(n \log n)$-time algorithm to determine whether any two disks in a set of $n$ intersect.

4. Delaunay Triangulation Problems: (20 = 5 + 15 points)
   A minimum weight triangulation of a set of points is a triangulation of the points that minimizes the sum of the distances of the edges in the triangulation.
   (a) Show an example of a set of points where the Delaunay triangulation and the minimum triangulation are the same.
   (b) Are the two triangulations always the same? Prove or disprove it.

5. Edge Flip Problem: (20 points)
   Show that any two triangulations of a convex polygon can be transformed into each other by edge flips.

6. Bonus Problem: (20 = 10 + 10 points are not included in the hundred percent of credit, but can make up the points you lose in all the 7 homework.)
   Let $K$ be a triangulation of a set of $n$ points in the plane. Let $l$ be a line that avoids all points.
   (a) Prove that $l$ intersects at most $2n - 4$ edges of $K$.
   (b) For every $n \geq 3$, give an example of $K$ and $l$ so that $l$ intersects exactly $2n - 4$ edges of $K$. 