L-system

L-systems are grammatical systems introduced by Lyndenmayer to describe biological developments such as the growth of plants and cellular organisms.

An L-system is composed of three parts $(\Sigma, h, w)$

- $\Sigma$: finite alphabet set of symbols
- $h$: rewriting rules each symbol is replaced by string of symbols
- $w$: axiom starting point

$h$ is finite substitutions, $h: \Sigma \to \Sigma^*$. 

$h(w)$ is computed by replacing every symbol in $w$ that has a rewrite rule by that rule.

A language $L$ of an L-system is the word sequence generated by

- $h^0(w) = w$
- $h^1(w) = h(w)$
- $h^2(w) = h(h(w))$
- $\ldots$

$L = \{h^i(w) | i \geq 0\}$

NOTE: If $h(a) = bb$ we will write this as a rule

$$a \Longrightarrow bb$$

meaning the symbol $a$ can be replaced by the symbols $bb$. 
Example:

\[\Sigma\] alphabet: \{a, b\}
h rules: 
\[a \rightarrow a\]
\[b \rightarrow ab\]
w axiom: ab

Notes:

- \(\rightarrow\) means “is replaced by”
- left hand side of rule must be a single character
- there is at most one rule for each character

What is the language \(L\) of strings represented by this L-system? that is, starting with the axiom, what are all the strings that can be generated from the rules?

\(L = \)

Example:

\[\Sigma\] alphabet: \{a\}
h rules: 
\[a \rightarrow aa\]
w axiom: a

\(L = \)

**Drawing a picture of an L-system**

Defining an L-system: (3 parts in this order)

- Axiom definition: This must be the first line of the file
- Production rules: Defines the replacement rules.
  - There must be spaces between the symbols on the right hand side of rules.
- Geometric rules: Defines colors, widths, etc.

Symbols for drawing and moving:

- g: draw a line one step in the current direction
- f: move forward one step in the current direction
Example:
This example is in the file: samp1

\[
\text{SET axiom } X
\]
\[
X \rightarrow g f g
\]
\[
\text{axiom definition}
\]
\[
\text{production rule}
\]
\[
\text{geometric rules}
\]
\[
\text{length of line drawn is 15 units}
\]
\[
\text{width of initial line is 5 units}
\]

Example:
This example is in the file: samp2

\[
\text{SET axiom } X
\]
\[
X \rightarrow g f g X
\]
\[
\text{start symbol is } X
\]
\[
\text{only change from previous program, repetition}
\]
\[
\text{length of line}
\]
\[
\text{width of initial line}
\]

L =
Symbols for changing direction

- +: change direction to the right in a determined angle
- -: change direction to the left in a determined angle
- &: change direction pitch down in a determined angle
- A: change direction pitch up in a determined angle
- x: change direction roll left in a determined angle
- /: change direction roll right in a determined angle
- %: change direction 180 degrees

Example:

This example is in the file: samp3

```
SET axiom X
X ==> g g g X + Y
Y ==> g
```

length of line
width of initial line
angle for change of direction
initial color

L =

Example:

We will make just a slight change in the L-system. This example is in the file: samp4
SET axiom X

\[
X \implies g \text{ change blue } g \text{ g change yellow } + \ Y \ X
\]
\[
Y \implies + \ Y
\]

only difference with file samp3

\[
\text{SET d 15}
\]

length of line

\[
\text{SET inwidth 5}
\]

width of initial line

\[
\text{SET angle 15}
\]

angle for change of direction

\[
\text{SET color black}
\]

initial color

L =

Stacking operations

- \[\text{: save in stack status of turtle which is current direction position and width of line}\]
- \[\text{: recover from stack status of turtle}\]

To make a branch, the turtle must draw one part of the branch and then come back to the fork position and draw the other part of the branch. Part of a string can be saved for processing by putting it within brackets \[\text{[ ]}\].

Example, consider the rules

SET axiom X

\[
X \implies g \text{ [ } \sim + \ Y \ g \text{ ] g}
\]

within \[\text{[ ]}\]'s is a branch

\[
Y \implies + \ Y
\]

the \(\sim\) means decrement the width of the line

\[
\text{SET d 18}
\]

increment for changing width of line

\[
\text{SET inwidth 4}
\]

\[
\text{SET inwidth 1}
\]

\[
\text{SET angle 30}
\]

\[
\text{SET color black}
\]

First string in L is \(g[\sim+Yg]g\). To draw this first draw the first line for the first \(g\). At this point, save the \([\sim+Yg]\) along with the current direction and the current width of the line. Continue drawing at the first symbol past the \[\text{[ ]}\]. Draw a line. Now that the end of the string has been reached, come back to the point in the drawing where the branch occurred and draw the string within the \[\text{[ ]}\]'s.

What is L? L =

\[
\text{[ ]}
\]

Suppose we change the X rule above by adding X onto the end:

\[
X \implies g \text{ [ } \sim + \ Y \ g \text{ ] g X}
\]
Now the L-system looks like:

Example of drawing plants via L-systems

Now we will examine some examples of growing plants.

Example:

This is in file: plant1

SET axiom X

X ==> g [ ~ + g Y ] g X
Y ==> g g [ ~ + g leaf ]
leaf ==> [ color Green { + f - f f - f + % + f - f f - f } ]

SET d 18
SET iniwidth 4
SET incwidth 1
SET angle 18
SET color black

Differences from previous examples:

- The color field allows you to specify a color for part of the drawing.
- The parenthesis { } are used to define a region that is to be filled in.

Example:
This is in file: plant2

SET axiom X

X ==> g [ ~ + g Y ] [ ~ = - g Y ] g X
Y ==> X g g [ ~ = + g leaf ]
leaf ==> [ color Green { + f - f f - f % + f - f f - f } ]

SET d 18
SET iniwidth 4
SET incwidth 1
SET angle 18
SET color black

Fractals

References:

- The Algorithmic Beauty of Plants, by P. Prusinkiewicz and A. Lindenmayer
- Automata, Languages, Development, by A. Lindenmayer and G. Rozenberg