Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:

Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms
Definition of TM

- **Storage**
  - tape

- **Actions**
  - write symbol
  - read symbol
  - move left (L) or right (R)

- **Computation**
  - initial configuration
    - start state
    - tape head on leftmost tape square
    - input string followed by blanks
  - processing computation
    - move tape head left or right
    - read from and write to tape
  - computation halts
    - final state

Formal Definition of TM

A TM M is defined by \( M=(Q,\Sigma,\Gamma,\delta,q_0,B,F) \) where

- \( Q \) is finite set of states
- \( \Sigma \) is input alphabet
- \( \Gamma \) is tape alphabet
- \( B \in \Gamma \) is blank
- \( q_0 \) is start state
- \( F \) is set of final states
- \( \delta \) is transition function

\( \delta(q,a) = (p,b,R) \) means “if in state q with the tape head pointing to an ’a’, then move into state p, write a ’b’ on the tape and move to the right”.

TM as Language recognizer

**Definition**: Configuration is denoted by ⊢.

If \( \delta(q,a) = (p,b,R) \) then a move is denoted

\[
abaqabba \vdash ababpbba
\]
**Definition:** Let M be a TM, $M=(Q, \Sigma, \Gamma, \delta, q_0, B, F)$. $L(M) = \{w \in \Sigma^*|q_0w \xrightarrow{*} x_1q_fx_2 \text{ for some } q_f \in F, \ x_1, x_2 \in \Gamma^* \}$

**TM as language acceptor**

M is a TM, w is in $\Sigma^*$,

- if $w \in L(M)$ then M halts in final state
- if $w \not\in L(M)$ then either
  - M halts in non-final state
  - M doesn’t halt

**Example**

$\Sigma = \{a, b\}$

Replace every second 'a' by a 'b' if string is even length.

- Algorithm
Example:

$L = \{a^n b^n c^n | n \geq 1 \}$

Is the following TM correct?

```
L = a; 1, R
2; 2, R
a; a, R
2; 2, R
b; 2, R
b; b, R
3; 3, R
b; b, R
c; 3, L
3; 3, L
a; a, L
b; b, L
2; 2, L
3; 3, L
```

**TM as a transducer**

TM can implement a function: $f(w) = w'$

**Definition:** A function with domain $D$ is *Turing-computable* or *computable* if there exists TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ such that

$$q_0 w \xrightarrow{*} q_f f(w)$$

$q_f \in F$, for all $w \in D$.

**Example:**

$f(x) = 2x$

$x$ is a unary number

```
start with: 111
↑
end with: 111111
↑
```
Is the following TM correct?

Example:

\[ L = \{ww | w \in \Sigma^+ \}, \Sigma = \{a, b\} \]