1. (3 pts) Prove that for every positive integer \( n \),
\[
1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \ldots + n(n+1)(n+2) = n(n+1)(n+2)(n+3)/4
\]

2. (3 pts) Prove that \( 3^n < n! \) if \( n \) is an integer greater than 6.

3. (3 pts) Prove that 3 divides \( n^3 + 2n \) whenever \( n \) is a positive integer.

4. (3 pts) Prove that if \( A_1, A_2, \ldots A_n \) and \( B \) are sets, then
\[
(A_1 - B) \cap (A_2 - B) \cap \ldots \cap (A_n - B) = (A_1 \cap A_2 \cap \ldots A_n) - B
\]

5. (5 pts) Let \( P(n) \) be the statement that a postage of \( n \) cents can be formed using just 4-cent and 7-cent stamps. The parts of this exercise outline a strong induction proof that \( P(n) \) is true for \( n \geq 18 \).

   (a) Show statements \( P(18), P(19), P(20) \) and \( P(21) \) are true, completing the basis step of the proof.
   
   (b) What is the inductive hypothesis of the proof?
   
   (c) What do you need to prove in the inductive step?
   
   (d) Complete the inductive step for \( k \geq 21 \).
   
   (e) Explain why these steps show that this statement is true whenever \( n \geq 18 \).

6. (3 pts) Use strong induction to show that every positive integer \( n \) can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers \( 2^0 = 1, 2^1 = 2, 2^2 = 4, \) and so on.

   Hint: For the inductive step, separately consider the case where \( k+1 \) is even and where it is odd. When it is even, note that \( (k+1)/2 \) is an integer.

7. (3 pts) For \( f(n+1) = f(n)f(n-1) \), find \( f(2), f(3), f(4), \) and \( f(5) \) if \( f \) is defined recursively by \( f(0) = f(1) = 1 \), for \( n = 1, 2, \ldots \).

8. (3 pts) Give a recursive definition of the set of positive integer powers of 3.

9. (3 pts) Recursively define the set of bit strings that have more zeros than ones.

10. (2 pts) A particular brand of shirt comes in 12 colors, has a male and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?

11. (6 pts) How many strings of four decimal digits
(a) do not contain the same digit twice?
(b) end with an even digit?
(c) have exactly three digits that are 9’s?

12. (8 pts) How many strings of eight uppercase letters are there

(a) if letters can be repeated?
(b) if no letter can be repeated?
(c) that start with X, if letters can be repeated?
(d) that start with X, if no letter can be repeated?

13. (6 pts) How many ways can a photographer at a wedding arrange 6 people in a row from a group of ten people, where the bride and groom are among these 10 people, if

(a) the bride must be in the picture?
(b) both the bride and groom must be in the picture?
(c) exactly one of the bride and the groom is in the picture?