1. The United States Postal Service (USPS) sells money orders identified by an 11-digit number $x_1x_2x_3\ldots x_{11}$. The first ten digits identify the money order; $x_{11}$ is a check digit that satisfies $x_{11} = x_1 + x_2 + \ldots x_{10} \mod 9$. Find the check digit for the USPS money order that has the identification number that starts with these ten digits: 755618873.

2. One digit in each of these identification numbers of a postal money order is smudged (see the information about USPS identification numbers in the previous problem). Can you recover the smudged digit indicated by Q, in each of these numbers?

(a) Q1223139784
(b) 6702120Q988

3. Encrypt the message "STOP POLLUTION" by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters. Use $f(p) = (p + 4) \mod 26$. 
4. Decrypt the following message that was encrypted using the Caesar cipher.

EOXH MHDQV

5. Suppose that the following ciphertext was produced by encrypting a plaintext message using a shift cipher. What is the original plaintext?

DVE CFMV KF NFEUVI, REU KYRK ZJ KYV JVVU FW JTZVETV

6. Use mathematical induction to prove the following. Prove that

\[ 2 - 2 \cdot 7 + 2 \cdot 7^2 - \ldots + 2 \cdot (-7)^n = \frac{1 - (-7)^{n+1}}{4} \]

whenever \( n \) is a nonnegative integer.

7. (a) Find a formula for \( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} \) by examining the values of this expression for small values of \( n \).

(b) Prove the formula you conjectured.
8. Prove that for every positive integer \( n \), \( \sum_{k=1}^{n} k2^k = (n - 1)2^{n+1} + 2 \)

9. For which nonnegative integers \( n \) is \( n^2 \leq n! \)? Prove your answer.

10. Prove that 21 divides \( 4^{n+1} + 5^{2n-1} \) whenever \( n \) is a positive integer.

11. Prove that if \( A_1, A_2, \ldots, A_n \) and \( B \) are sets, then \( (A_1 \cap A_2 \cap \ldots \cap A_n) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_n \cup B) \)
12. Prove or disprove that a checkerboard with shape $6 \times 2^n$ can be completely covered using right triominoes whenever $n$ is a positive integer.

13. (a) Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps.
   (b) Prove your answer to a) using the principle of mathematical induction. Be sure to state explicitly your inductive hypothesis in the inductive step.
   (c) Prove your answer to a) using strong induction. How does the inductive hypothesis in this proof differ from that in the inductive hypothesis for a proof using mathematical induction?

14. Prove that $f_1^2 + f_2^2 + \ldots + f_n^2 = f_n f_{n+1}$ where $f_n$ is the $n$th Fibonacci number.