Relational Database Design Theory
Part II

CPS 116
Introduction to Database Systems

Announcements

- Homework #2 sample solution available
- Project milestone #1 due today (Sep. 30)
- Midterm next Thursday in class
  - Open book, open notes
  - Sample midterm (from last year) available
    - Solution available next Tuesday

Review

- Functional dependencies
  - $X \rightarrow Y$: If two rows agree on $X$, they must agree on $Y$
    - A generalization of the key concept
  - Non-key functional dependencies: a source of redundancy
    - Non-trivial $X \rightarrow Y$ where $X$ is not a superkey
      - Called a BCNF violation
  - BCNF decomposition: a method for removing redundancies
    - Given $R(X, Y, Z)$ and a BCNF violation $X \rightarrow Y$, decompose $R$ into $R_1(X, Y)$ and $R_2(X, Z)$
      - A lossless join decomposition
  - Schema in BCNF has no redundancy due to FD's
3NF (BCNF is too much)
Multivalued dependencies: another source of redundancy
4NF (BCNF is not enough)

Motivation for 3NF

Address (street_address, city, state, zip)
  • street_address, city, state → zip
  • zip → city, state

Keys
  • {street_address, city, state}
  • {street_address, zip}

BCNF?
  • Violation: zip → city, state

To decompose or not to decompose

Address₁ (zip, city, state)
Address₂ (street_address, zip)

FD’s in Address₁

FD’s in Address₂

Hey, where is street_address, city, state → zip?
  • Cannot check without joining Address₁ and Address₂ back together

Problem: Some lossless join decomposition is not dependency-preserving

Dilemma: Should we get rid of redundancy at the expense of making constraints harder to enforce?
**3NF**

- **R** is in Third Normal Form (3NF) if for every non-trivial FD $X \rightarrow A$ (where $A$ is a single attribute), either
  - $X$ is a superkey of $R$, or
  - $A$ is a member of at least one key of $R$
- Intuitively, BCNF decomposition on $X \rightarrow A$ would “break” the key containing $A$
- So **Address** is already in 3NF
- **Tradeoff:**
  - Can enforce all original FD’s on individual decomposed relations
  - Might have some redundancy due to FD’s

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**BNCF = no redundancy?**

- **Student** ($SID$, $CID$, club)
  - Suppose your classes have nothing to do with the clubs you join
  - FD’s?
    
    | SID | CID | club  |
    |-----|-----|-------|
    | 150 | 75116 | ballet |
    | 152 | 75116 | sumo   |
    | 142 | 55114 | ballet |
    | 142 | 55114 | sumo   |
    | 123 | 55114 | chess  |
    | 123 | 55114 | golf   |

- **BNCF?**

- **Redundancies?**

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**Multivalued dependencies**

- A multivalued dependency (MVD) has the form $X \rightarrow Y$, where $X$ and $Y$ are sets of attributes in a relation $R$
- $X \rightarrow Y$ means that whenever two rows in $R$ agree on all the attributes of $X$, then we can swap their $Y$ components and get two new rows that are also in $R$

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>b1</td>
<td>c1</td>
</tr>
<tr>
<td>a2</td>
<td>b2</td>
<td>c2</td>
</tr>
<tr>
<td>a3</td>
<td>b3</td>
<td>c3</td>
</tr>
<tr>
<td>a4</td>
<td>b4</td>
<td>c4</td>
</tr>
</tbody>
</table>

Must be in $R$ too
MVD examples

*Student (SID, CID, club)*

- SID → CID
  - Intuition:
  - SID, CID → club
  - SID, CID → SID

Complete MVD + FD rules

- FD reflexivity, augmentation, and transitivity
- MVD complementation:
  If X → Y, then X → attr(R) – X – Y
- MVD augmentation:
  If X → Y and V ⊆ W, then XW → YY
- MVD transitivity:
  If X → Y and Y → Z, then X → Z – Y
- Replication (FD is MVD):
  If X → Y, then X → Y
  Try proving things using these!
- Coalescence:
  If X → Y and Z ⊆ Y and there is some W disjoint from Y such that W → Z, then X → Z

An elegant solution: chase

- Given a set of FD’s and MVD’s D, does another dependency d (FD or MVD) follow from D?
- Procedure
  - Start with the hypothesis of d, and treat them as “seed” tuples in a relation
  - Apply the given dependencies in D repeatedly
    - If we apply an FD, we infer equality of two symbols
    - If we apply an MVD, we infer more tuples
    - If we infer the conclusion of d, we have a proof
    - Otherwise, if nothing more can be inferred, we have a counterexample
Proof by chase

In \( R(A, B, C, D) \), does \( A \rightarrow B \) and \( B \rightarrow C \) imply that \( A \rightarrow C \)?

<table>
<thead>
<tr>
<th>Have</th>
<th>Need</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \rightarrow B )</td>
<td>( A \rightarrow B )</td>
</tr>
<tr>
<td>( B \rightarrow C )</td>
<td>( B \rightarrow C )</td>
</tr>
</tbody>
</table>

Another proof by chase

In \( R(A, B, C, D) \), does \( A \rightarrow B \) and \( B \rightarrow C \) imply that \( A \rightarrow C \)?

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<tr>
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<th>Need</th>
</tr>
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<tbody>
<tr>
<td>( A \rightarrow B )</td>
<td>( A \rightarrow B )</td>
</tr>
<tr>
<td>( B \rightarrow C )</td>
<td>( B \rightarrow C )</td>
</tr>
</tbody>
</table>

In general, both new tuples and new equalities may be generated.

Counterexample by chase

In \( R(A, B, C, D) \), does \( A \rightarrow BC \) and \( CD \rightarrow B \) imply that \( A \rightarrow B \)?

<table>
<thead>
<tr>
<th>Have</th>
<th>Need</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \rightarrow BC )</td>
<td>( A \rightarrow BC )</td>
</tr>
</tbody>
</table>
4NF

- A relation \( R \) is in Fourth Normal Form (4NF) if
  - For every non-trivial MVD \( X \rightarrow Y \) in \( R \), \( X \) is a superkey
  - That is, all FD's and MVD's follow from "key \( \rightarrow \) other attributes" (i.e., no MVD's, and no FD's besides key functional dependencies)

- 4NF is stronger than BCNF
  - Because every FD is also a MVD

4NF decomposition algorithm

- Find a 4NF violation
  - A non-trivial MVD \( X \rightarrow Y \) in \( R \) where \( X \) is not a superkey
- Decompose \( R \) into \( R_1 \) and \( R_2 \), where
  - \( R_1 \) has attributes \( X \cup Y \)
  - \( R_2 \) has attributes \( X \cup Z \) (\( Z \) contains attributes not in \( X \) or \( Y \))
- Repeat until all relations are in 4NF

- Almost identical to BCNF decomposition algorithm
- Any decomposition on a 4NF violation is lossless

4NF decomposition example

- Student (\( SID, CID, club \))
  - 4NF violation: \( SID \rightarrow CID \)
- Enroll (\( SID, CID \))
- Join (\( SID, club \))
  - 4NF

\[
\begin{align*}
\text{ID} & \quad \text{CID} & \quad \text{Join} \\
142 & \quad CPS116 & \quad \text{ballet} \\
142 & \quad CPS116 & \quad \text{sumo} \\
142 & \quad CPS114 & \quad \text{ballet} \\
142 & \quad CPS114 & \quad \text{sumo} \\
123 & \quad CPS116 & \quad \text{golf} \\
123 & \quad CPS116 & \quad \text{golf} \\
123 & \quad CPS116 & \quad \text{golf} \\
\end{align*}
\]
3NF, BCNF, 4NF, and beyond

<table>
<thead>
<tr>
<th>Anomaly/normal form</th>
<th>3NF</th>
<th>BCNF</th>
<th>4NF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lose FD’s?</td>
<td>No</td>
<td>Possible</td>
<td>Possible</td>
</tr>
<tr>
<td>Redundancy due to FD’s</td>
<td>Possible</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Redundancy due to MVD’s</td>
<td>Possible</td>
<td>Possible</td>
<td>No</td>
</tr>
</tbody>
</table>

- **Of historical interests**
  - **1NF**: All column values must be atomic
  - **2NF**: There is no partial functional dependency (a non-trivial FD $X \rightarrow A$ where $X$ is a proper subset of some key)

**Summary**

- **Philosophy behind BCNF, 4NF**: Data should depend on the key, the whole key, and nothing but the key!
- **Philosophy behind 3NF**: … But not at the expense of more expensive constraint enforcement!