Homework Assignment 1

Homework on Sorting

Write the solution to each problem on a single page.
The deadline for handing in solutions is September 15.

Problem 1. (20 points). Give an algorithm that merges $k$ sorted lists with a total of $n$ items into a single sorted list and does this in time $O(n \log k)$.

Problem 2. (20 points). Draw the decision tree for heap-sort applied to sorting a list of three distinct items, $a_1, a_2, a_3$.

Problem 3. (20 = 13 + 7 points). Consider a permutation of $\{1, 2, \ldots, n\}$ and let $\pi(k)$ denote the position of item $k$ in the permutation. Then $D = \frac{1}{n} \sum_{k=1}^{n} |\pi(k) - k|$ is the average distance that a number travels during sorting.

(a) What is the expected value of $D$ if the permutation is chosen at random?
(b) What does the answer to (a) say about sorting algorithms where only adjacent items are interchanged?

Problem 4. (20 = 10 + 10 points). Consider distinct items $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = 1.0$. The weighted median is the item $x_k$ that satisfies

$$\sum_{x_i < x_k} w_i < 0.5 \text{ and } \sum_{x_j > x_k} w_j \leq 0.5.$$

(a) Show how to compute the weighted median of $n$ items in worst-case time $O(n \log n)$ using sorting.
(b) Show how to compute the weighted median in worst-case time $O(n)$ using a linear-time median algorithm.

Problem 5. (20 = 10 + 10 points). Consider a lazy version of heapsort where each item in the heap is either smaller than or equal to every other item in its subtree, or the item is identified as uncertified. To certify an item, we certify its children and then exchange it with the smaller child provided it is smaller than the item itself. Suppose $A[1..n]$ is a heap with all items uncertified.

(a) How much time does it take to certify $A[1]$?
(b) Does certifying $A[1]$ turn $A$ into a proper heap in which every item satisfies the heap property? (Justify your answer.)