# Homework on Searching

Write the solution to each problem on a single page of a separate sheet of paper. The deadline for handing in solutions is September 29.

**Problem 1.** (20 points). Suppose you have an infinite array $A[1..]$ with items sorted in non-increasing order. Describe an $O(\log n)$-time algorithm to find the smallest index $n$ such that $A[n] < 0$. (An $O(n)$-time algorithm for this problem is trivial, scanning the array from the beginning until it find the first negative item.)

**Problem 2.** (20 = 6 + 6 + 8 points). Write recursive traversal functions that compute the following measures of a binary tree with root $g$:

(a) the size;
(b) the height;
(c) the path length, which is the sum of depths of all nodes.

**Problem 3.** (20 = 10 + 10 points). Let $g$ be the root of a binary tree.

(a) Write recursive functions $\text{PRE}$ and $\text{POST}$ that compute for each node $\mu$ its positions $\mu \rightarrow \text{pre}$ and $\mu \rightarrow \text{post}$ in the preorder and the postorder sequences of the nodes.
(b) Prove that $\mu$ is a proper ancestor of $\nu$ iff $\mu \rightarrow \text{pre} < \nu \rightarrow \text{pre}$ and $\mu \rightarrow \text{post} > \nu \rightarrow \text{post}.$

**Problem 4.** (20 = 5 + 5 + 5 + 5 points). Equal keys pose a problem for the implementation of binary search trees.

(a) What is the asymptotic performance of $\text{TREE-INSERT}$ on page 261 of CLRS when used to insert $n$ items with identical keys into an initially empty binary search tree?

We propose to improve $\text{TREE-INSERT}$ by testing before Line 11 whether or not $\text{key}[z] = \text{key}[y]$. If equality holds, we implement one of the following strategies. For each strategy, find the asymptotic performance of inserting $n$ items with identical keys into an initially empty binary search tree. (The strategies are described for Line 5.)

(b) Keep a boolean flag $b[x]$ at node $x$ and set $x$ to either $\text{left}[x]$ or $\text{right}[x]$ based on the value of $b[x]$, which alternates between TRUE and FALSE each time $x$ is visited during insertion of a node with the same key as $x$.
(c) Keep a list of nodes with equal keys at $x$, and insert $z$ into this list.
(d) Randomly set $x$ to either $\text{left}[x]$ or $\text{right}[x]$.

**Problem 5.** (20 = 8 + 4 + 8 points). Let us ignore the existence of leap years and assume the probability that a person is born on any one day in the year is $\frac{1}{365}$.

(a) In a room of $k$ people, what is the probability that at least two have the same birthday?
(b) Use your calculator to compute the probabilities for $k = 10, 20, 30$.
(c) What is the smallest $k$ such that the probability that all $k$ people have different birthdays is less than one half?