1. A connected graph is vertex biconnected if there is no vertex whose removal disconnects the graph. A connected graph is edge biconnected if there is no edge whose removal disconnects the graph. Give a proof or counterexample for each of the following statements (consider only the undirected graphs having three or more vertices): (25 points)
   a) A vertex biconnected graph is edge biconnected.
   b) An edge biconnected graph is vertex biconnected.

2. Suppose you are given a directed graph \( G = (V, E) \) with a weight assigned to each vertex. Call this weight \( w(v) \). In this graph, the directed arc \((u, v)\) is present if and only if \( w(u) \leq w(v) \). For example, there would be an arc from a vertex with weight 8 to vertices of weight 10 or 15, but not to vertices of weight 2, 3, or 6. Assume all weights are distinct, i.e. there are no vertices with identical weights. (25 points)
   a) We call a graph transitive if for all vertices \( u, v, w \), if there are two edges \((u, v)\) and \((v, w)\), then the edge \((u, w)\) is present. Prove this graph is transitive.
   b) Show that this graph has no cycles, meaning it’s a DAG.
   c) Find an upper bound (Big-O notation) on the number of edges in the graph, if there are \( n \) vertices.
   d) Devise a sorting algorithm that employs this graph to sort the numbers. Your algorithm should be linear in the number of edges. Is this a good way to sort numbers? Why or why not?