On homework, you may discuss with other students in the course about how to solve a problem, but the write-up should be your own. You must include the names of any students you consulted with. Give credit where credit is due.

1. (5 pts) Given the sets of integers below, describe the new sets created using similar notation.
   \[ S_1 = \{ 0, 4, 8, 12, \ldots \} \]
   \[ S_2 = \{ n > 0 \mid n \text{ is divisible by } 5 \} \]
   \[ S_3 = \{ n > 0 \mid n \text{ is even} \} \]
   \[ S_4 = \{ 3, 5 \} \]
   \[ S_5 = \{ 1, 2, 3, 4, 5, 6 \} \]
   (a) \( S_1 \cap S_2 = \)
   (b) \( S_3 \cap S_4 = \)
   (c) \( S_1 \cup S_3 = \)
   (d) \( S_5 - S_4 = \)
   (e) \( S_3 \times S_4 = \)

2. (3 pts) True or False?
   (a) \( \emptyset \subseteq \{x, y, \{x, y\}\} \)
   (b) \( \{x, y\} \subseteq \{x, y, \{x, y\}\} \)
   (c) \( \{x\} \in \{x, y, \{x, y\}\} \)

3. (4 pts) Prove by induction \( 1 + 3 + 5 + 7 + \ldots + (2n - 1) = n^2 \) for all \( n > 0 \). Show all steps (basis, IH, and IS).

4. (4 pts) Prove by induction \( 2^n < n! \), \( n \geq 4 \)

5. (4 pts) Let \( n > 0 \). Prove any \( 2^n \times 2^n \) chessboard with one square removed can be completely covered by L-shaped tiles, where each tile covers 3 squares. Show all steps (basis, IH and IS). An L-shaped tile looks like:

   \[
   \begin{array}{|ccc|}
   \hline
   \hline
   \end{array}
   \]
6. (5 pts) Given the languages below, describe the new languages created using the simplest notation.

\[ \Sigma = \{a, b, c\} \]
\[ L_1 = \{b^n \mid n \geq 1\} \]
\[ L_2 = \{ba^n \mid n \geq 0\} \]
\[ L_3 = \{b^n a^n \mid n > 0\} \]

(a) \( L_1 \cap \Sigma^* = \)
(b) \( L_2 \cap L_3 = \)
(c) \( L_1 \circ L_1 = \)
(d) \( L_2 \circ L_2 = \)
(e) \( L_2 \circ L_2^R = \)

7. (10 pts) Consider each of the following languages.

(a) \( L = \{b, ab, bab\} \)
   i. write a grammar that generates the language.

(b) \( L = \{a^{2n}b^n \mid n > 0\} \)
   i. list 3 strings in the language
   ii. write a grammar that generates the language.

(c) \( L = \{a^m b^n c^m \mid n > 0, m \geq 0\} \)
   i. list 3 strings in the language
   ii. write a grammar that generates the language.

(d) \( \Sigma = \{a, b\}, L = \{w \in \Sigma^* \mid n_a(w) = 2\}, (n_a(w) \text{ means number of } a's \text{ in } w) \)
   i. list 3 strings in the language
   ii. write a grammar that generates the language.