Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines

1. Given Turing Machines M1 and M2
   Notation for
   - Run M1
   - Run M2

2. Given Turing Machines M1 and M2
   Notation for
   - Run M1
   - If x is current symbol
     - then Run M2
3. Given Turing Machines M1, M2, and M3

Notation for

- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

More Notation for Simplifying Turing Machines

Suppose \( \Gamma = \{a, b, c, B\} \)

- \( z \) is any symbol in \( \Gamma \)
- \( x \) is a specific symbol from \( \Gamma \)

1. \( s \) - start
2. \( R \) - move right
3. **L** - move left

4. **x** - write x (and don’t move)

5. **Ra** - move right until you see an **a**

6. **La** - move left until you see an **a**

7. **R¬a** - move right until you see anything that is not an **a**

8. **L¬a** - move left until you see anything that is not an **a**

9. **h** - halt in a final state

10. \[ a, b \rightarrow w \]

If the current symbol is a or b, let w represent the current symbol.
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb

input: ba, output: ba

What is the running time?
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}, |w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $abbabb$, output: $abbabbb$

The tape head should finish pointing at the leftmost symbol of $w$.

Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function $f:D \rightarrow \mathbb{R}$ is a TM $M$, which given input $d \in D$, halts with answer $f(d) \in \mathbb{R}$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

\[
\begin{array}{c}
\text{start with:} & 111+1111 \\
& \uparrow \\
\text{end with:} & 1111111 \\
& \uparrow 
\end{array}
\]
**Example:** Copy a String, \( f(w) = w0w, \ w \in \Sigma^*, \ \Sigma = \{a, b, c\} \)

Denoted by \( C \)

- start with: \( abac \)
- end with: \( abac0abac \)

**Algorithm:**

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol

\[
\text{Algorithm:}
\]

\[
\text{\begin{array}{c}
\text{s} & \text{R} & 0 & \text{L} & \text{R} & \{a, b, c\} \\
\text{B} & \text{B} & \text{w} & \text{B} & \text{R} & \text{w} & \text{L} & \text{w} \\
\end{array}}
\]

- make a copy of the symbol

\[
\text{Algorithm:}
\]

\[
\text{\begin{array}{c}
\text{s} & \text{R} & 0 & \text{L} & \text{R} & \{a, b, c\} \\
\text{B} & \text{B} & \text{w} & \text{B} & \text{R} & \text{w} & \text{L} & \text{w} \\
\end{array}}
\]

- make a copy of the symbol
**Example:** Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

![Diagram of tape head movement](image)

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
**Example:** Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: babcaBba

$\uparrow$

end with: bacaBBba

$\uparrow$

(similar to $S_R$)
**Example:** Add unary numbers

This time use shift.

**Example:** Multiply two unary numbers, $f(x\cdot y)=x\cdot y$, $x$ and $y$ unary numbers. Assume $x,y>0$.

```
start with:     1111*11
               ↑

dend with:     11111111
               ↑
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